Matrix Completion goes Rogue (nonlinear)!

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Textbook Example
Recommender Systems
Comedy
Romance
Action
Horror

= a + b + c + d
Column lies in a subspace!
We want to find this Subspace!
We want to find this Subspace!  
Problem is: data is incomplete!
We want to find this Subspace!

Problem is: data is incomplete!
What if dependencies are nonlinear?! Problem is: data is incomplete!

Columns lie in a variety!
What if dependencies are nonlinear?!

Problem is: data is incomplete!
What am I telling you?
Given: Incomplete data matrix

Goal: Find linear subspace that explains data

Low-Rank Matrix Completion
Goal: Find linear subspace that explains data

Given: Incomplete data matrix

Low-Rank Matrix Completion

Algebraic Variety!
Is it possible?  When?  How?
Given: Incomplete data matrix

Goal: Find linear subspace that explains data

Algebraic Variety!

Is it possible?

When?

How?
Given: Incomplete data matrix

Goal: Find linear subspace that explains data

Algebraic Variety!

Is it possible? When? How?
Given: Incomplete data matrix

Goal: Find linear subspace that explains data

Algebraic Variety!

Is it possible? Yes

When?

How?
**Goal:** Find linear subspace that explains data

**Given:** Incomplete data matrix

**Is it possible?** Yes

**When?** When you observe the right entries

**How?**
Given: Incomplete data matrix

Goal: Find linear subspace that explains data

X ∈ Algebraic Variety!

Is it possible? Yes

When? When you observe the right entries

How? Using tensors
ROLL UP YOUR SLEEVES!
Consider a point in a variety

\[ \{ f_1(x) = 0, f_2(x) = 0, \ldots, f_N(x) = 0 \} \]

\[ x \in \mathbb{R}^d \]
Consider a point in a variety $\mathbf{x} \in \mathbb{R}^d$ that satisfies the following system of equations:

\[
\begin{align*}
    v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 &= 0 \\
v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 &= 0 \\
    \vdots & \\
v_{N1}x_1^2 + v_{N2}x_1x_2 + v_{N3}x_1x_3 + \cdots + v_{ND}x_d^2 &= 0
\end{align*}
\]
\[ \mathbf{x} \in \mathbf{v} \quad \sum_{i=1}^{D} v_{1i} x_i^2 + v_{12} x_1 x_2 + v_{13} x_1 x_3 + \cdots + v_{1d} x_d^2 = 0 \]
$\mathbf{x} \in \mathcal{V} \quad \begin{cases} v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 = 0 \\ v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 = 0 \end{cases}$

\[
\begin{bmatrix}
v_{11} & v_{12} & v_{13} & \cdots & v_{1D} \\
v_{21} & v_{22} & v_{23} & \cdots & v_{2D}
\end{bmatrix}
\begin{bmatrix}
x_1^2 \\
x_1x_2 \\
x_1x_3 \\
\vdots \\
x_d^2
\end{bmatrix} = \mathbf{0}
\]
\[
\mathbf{x} \in \mathcal{V} \quad \left\{ \begin{array}{l}
v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 = 0 \\
v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 = 0 \\
\vdots \\
v_{N1}x_1^2 + v_{N2}x_1x_2 + v_{N3}x_1x_3 + \cdots + v_{ND}x_d^2 = 0
\end{array} \right.
\]

\[
\begin{bmatrix}
v_{11} & v_{12} & v_{13} & \cdots & v_{1D} \\
v_{21} & v_{22} & v_{23} & \cdots & v_{2D} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
v_{N1} & v_{N2} & v_{N3} & \cdots & v_{ND}
\end{bmatrix}
\begin{bmatrix}
x_1^2 \\
x_1x_2 \\
x_1x_3 \\
\vdots \\
x_d^2
\end{bmatrix}
= \mathbf{0}
\]
\[ \mathbf{x} \in \mathcal{V} \]

\[
\begin{align*}
\mathbf{v}_1 x_1^2 + \mathbf{v}_2 x_1 x_2 + \mathbf{v}_3 x_1 x_3 + \cdots + \mathbf{v}_{1D} x_d^2 &= 0 \\
\mathbf{v}_1 x_1^2 + \mathbf{v}_2 x_1 x_2 + \mathbf{v}_3 x_1 x_3 + \cdots + \mathbf{v}_{2D} x_d^2 &= 0 \\
&\vdots \\
\mathbf{v}_1 x_1^2 + \mathbf{v}_2 x_1 x_2 + \mathbf{v}_3 x_1 x_3 + \cdots + \mathbf{v}_{ND} x_d^2 &= 0
\end{align*}
\]

\[
\begin{bmatrix}
\mathbf{v}_{11} & \mathbf{v}_{12} & \mathbf{v}_{13} & \cdots & \mathbf{v}_{1D} \\
\mathbf{v}_{21} & \mathbf{v}_{22} & \mathbf{v}_{23} & \cdots & \mathbf{v}_{2D} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{v}_{N1} & \mathbf{v}_{N2} & \mathbf{v}_{N3} & \cdots & \mathbf{v}_{ND}
\end{bmatrix}
\begin{bmatrix}
x_1^2 \\
x_1 x_2 \\
\vdots \\
x_1 x_3 \\
\vdots \\
x_d^2
\end{bmatrix}
= 0
\]

\[
\mathbf{V}^T \mathbf{x} \otimes 2 = 0
\]
$x \in \mathcal{V} \iff V^T x \otimes^2 = 0$
\[ x \in \mathcal{N} \quad \iff \quad V^T x \otimes^2 = 0 \quad \iff \quad x \otimes^2 \in \ker V^T \]
\[ x \in \mathcal{V} \iff V^T x \otimes^2 = 0 \]

\[ \iff \quad x \otimes^2 \in \ker V^T \]

\[ \| \quad S \]
\[
V^T x \otimes^2 = 0
\]
\[
x \otimes^2 \in \ker V^T
\]
\[
x \otimes^2 \in S
\]
$x \in \mathcal{V} \iff V^T x \otimes 2 = 0$

$\iff x \otimes 2 \in \ker V^T$

$\iff x \otimes 2 \in \mathcal{S}$
$x \in \mathcal{V} \iff V^T x \otimes^2 = 0$

$\iff x \otimes^2 \in \ker V^T$

$\iff x \otimes^2 \in \mathcal{S}$

Variety = Tensorized Subspace!
Variety \( \mathcal{V} \)
What does this mean?
What does this mean?

Algebraic Variety (nonlinear) \( X \in \mathcal{V} \) \( \xrightarrow{\sim} \) Tensorized Subspace (linear) \( X \otimes^2 \in \mathcal{S} \)
What does this mean?

Algebraic Variety (nonlinear) \[ X \in \mathcal{V} \]

Tensorized Subspace (linear) \[ X \otimes^2 \in S \]

Find Subspace, Find Variety!
What does this mean?
What does this mean?

Is this just standard Low-Rank Matrix Completion?
What does this mean?

Is this just standard Low-Rank Matrix Completion?

More or less…
Recall...
Recall...
Recall…

\[ x \in \mathcal{V} \]

\[ x \otimes^2 \in \mathcal{S} \]

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  x_1 x_1 \\
  x_1 x_2 \\
  x_1 x_3 \\
  x_2 x_1 \\
  x_2 x_2 \\
  x_2 x_3 \\
  x_3 x_1 \\
  x_3 x_2 \\
  x_3 x_3 \\
\end{bmatrix}
\]
Recall…
Recall...
Recall...
Recall…
The sampling is highly restricted!
The sampling is highly restricted!
The sampling is highly restricted!
The sampling is highly restricted!
Small letters in LRMC: Incoherence and Uniform Sampling
In general

Given: available samplings
In general

Given: available samplings
In general

Given: available samplings

Can we find $S$?
In general

Given: available samplings

Can we find $S$?

**Theorem** (P.-A., Ongie, Balziano, Willett, Nowak, 2017)

Suppose $V$ is in general position. With probability 1, $S$ can be uniquely recovered if and only if there is a matrix $\Omega \otimes 2$ formed with $D-R$ columns of $\Omega \otimes 2$ such that every $\Omega_\ell \otimes 2$ formed with a subset of columns in $\Omega \otimes 2$ satisfies:

$$\#\text{rows}\_\text{with}\_\text{observations}(\Omega_\ell \otimes 2) \geq \#\text{columns}(\Omega_\ell \otimes 2) + R.$$

Furthermore, this condition is true if and only if $\dim \ker A^T = R$, whence $S = \ker A^T$. 
Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathcal{V}$ is in general position. With probability 1, $\mathcal{S}$ can be uniquely recovered if and only if there is a matrix $\Omega^\otimes 2$ formed with $D-R$ columns of $\Omega^\otimes 2$ such that every $\Omega^\otimes 2_\ell$ formed with a subset of columns in $\Omega^\otimes 2$ satisfies:

$$\#\text{rows_with_observations}(\Omega^\otimes 2_\ell) \geq \#\text{columns}(\Omega^\otimes 2_\ell) + R.$$  

Furthermore, this condition is true if and only if $\dim \ker A^T = R$, whence $\mathcal{S} = \ker A^T$.

$$D=5,\ R=1$$

$\Omega^\otimes 2 = \begin{array}{ccc} \text{Good} \\
\end{array}$

$\Omega^\otimes 2 = \begin{array}{ccc} \text{Bad} \\
\end{array}$

$3 < 3 + 1$
Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose \( \mathcal{V} \) is in general position. With probability 1, \( S \) can be uniquely recovered if and only if there is a matrix \( \Omega_\star ^\times^2 \) formed with D-R columns of \( \Omega ^\times^2 \) such that every \( \Omega_\ell ^\times^2 \) formed with a subset of columns in \( \Omega_\star ^\times^2 \) satisfies:

\[
\text{#rows_with_observations}(\Omega_\ell ^\times^2) \geq \text{#columns}(\Omega_\ell ^\times^2) + R.
\]

Furthermore, this condition is true if and only if \( \dim \ker A^T = R \), whence \( S = \ker A^T \).

In words:

- Yes, it is possible to find the subspace \( S \).
- Iff you observe the right entries (rows vs cols condition).
- There is an easy way to check this rows vs cols condition.
- If the condition is satisfied, there is an easy way to find \( S \).
Data $\in \mathcal{S}$

$$X \otimes^2$$

$\geq R$

Projection

$\mathcal{S}_1$

A Flavor of our Ideas

Data $\sim$ Projection
Data Projection

\[ X \otimes^2 \leq R \]

\[ S_1 \]

\[ S_1 \]

A Flavor of our Ideas

Data \sim Projection
How is this information useful?
How is this information useful?
How is this information useful?
How is this information useful?
The projection $S_1$ imposes one restriction on what $S$ may be.

How is this information useful?
If we observe more projections...
If we observe more projections...
If we observe more projections…

We get more restrictions…
If we observe more projections...
We get more restrictions...
If we observe more projections...

We get more restrictions...
If we observe more projections…
Can we find the whole subspace?
If we observe more projections...

Can we find the whole subspace?

Sometimes you can, sometimes you can’t.
If we observe more projections…

Can we find the whole subspace?

Sometimes you can, sometimes you can’t.

Depends on which projections you get.
If we observe more projections...

Can we find the whole subspace?

Sometimes you can, sometimes you can't.

Depends on which projections you get.
If we observe more projections...

Can we find the whole subspace?

Sometimes you can, sometimes you can’t. Depends on which projections you get. You need to observe the right projections.

Some restrictions may be redundant!
Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathcal{V}$ is in general position. With probability 1, $S$ can be uniquely recovered if and only if there is a matrix $\Omega \otimes^2$ formed with $D-R$ columns of $\Omega \otimes^2$ such that every $\Omega_{\ell} \otimes^2$ formed with a subset of columns in $\Omega \otimes^2$ satisfies:

$$\#\text{rows with observations}(\Omega_{\ell} \otimes^2) \geq \#\text{columns}(\Omega_{\ell} \otimes^2) + R.$$ 

Furthermore, this condition is true if and only if $\dim \ker A^T = R$, whence $S = \ker A^T$.

$D=5$, $R=1$

$\Omega \otimes^2 = \begin{bmatrix}
\text{Good}
\end{bmatrix}$

$\Omega \otimes^2 = \begin{bmatrix}
\text{Bad}
\end{bmatrix}$
**Theorem** (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathcal{V}$ is in general position. With probability 1, $\mathcal{S}$ can be uniquely recovered if and only if there is a matrix $\Omega \otimes^2$ formed with $D-R$ columns of $\Omega \otimes^2$ such that every $\Omega_{\ell} \otimes^2$ formed with a subset of columns in $\Omega \otimes^2$ satisfies:

$$\#\text{rows}_\text{with}_\text{observations}(\Omega_{\ell} \otimes^2) \geq \#\text{columns}(\Omega_{\ell} \otimes^2) + R.$$ 

Furthermore, this condition is true if and only if $\dim \ker A^T = R$, whence $\mathcal{S} = \ker A^T$. 

---

$D=5, R=1$

---

$\Omega \otimes^2 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$

$\Omega \otimes^2 = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 & 5
\end{bmatrix}$

Indicates coordinates involved in given projections

**Good**

**Bad**
**Theorem** (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathcal{V}$ is in general position. With probability 1, $\mathcal{S}$ can be uniquely recovered if and only if there is a matrix $\Omega^{\otimes 2}$ formed with D-R columns of $\Omega^{\otimes 2}$ such that every $\Omega^{\otimes 2}_\ell$ formed with a subset of columns in $\Omega^{\otimes 2}$ satisfies:

$$\#\text{rows}_\text{with}_\text{observations}(\Omega^{\otimes 2}_\ell) \geq \#\text{columns}(\Omega^{\otimes 2}_\ell) + R.$$ 

Furthermore, this condition is true if and only if $\dim \ker A^T = R$, whence $\mathcal{S} = \ker A^T$.

D=5, R=1

Encodes information of given projections

Indicates coordinates involved in given projections

Good

Bad
So, what do we know so far?
So, what do we know so far?
So, what do we know so far?

We need to observe the right entries

Find Subspace, Find Variety!

Algebraic Variety (nonlinear)

Tensorized Subspace (linear)
So, what do we know so far?

Is it even possible that these produce the right entries?

We need to observe the right entries

Find Subspace, Find Variety!

Algebraic Variety (nonlinear)

Tensorized Subspace (linear)
So, what do we know so far?

Is it even possible that these produce the right entries? **Yes :)**

We need to observe the right entries

Find Subspace, Find Variety!

Algebraic Variety (nonlinear)

Tensorized Subspace (linear)
Subspace Dimension = $R$
Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathbf{V}$ is in general position. Suppose each column $\mathbf{x}$ has $m$ samples.

(i) If $m < r$, then $\mathcal{S}$ cannot be uniquely determined.

(ii) There are cases with $m = r$ and $m = r+1$ where $\mathcal{S}$ cannot be uniquely determined.

(iii) If $m \geq r+2$, then $\mathcal{S}$ can be uniquely determined (if you observe the right entries).
So, what do we know so far?
So, what do we know so far?
So, what do we know so far?

We can find \( S = \ker A^T \)
So, what do we know so far?
So, what do we know so far?
So, what do we know so far?

(Provable Algorithm)
What is this good for?
What is this good for?

Unions of Subspaces!
What is this good for?

Unions of Subspaces!
Multiple subspaces

We want to find them all!
Multiple subspaces
We want to find them all!
Multiple subspaces

We want to find them all!
Multiple subspaces

We want to find them all!
Multiple subspaces

We want to find them all!
Multiple subspaces
We want to find them all!
Multiple subspaces

We want to find them all!
Multiple subspaces

We want to find them all!
Multiple subspaces

We want to find them all!
Things get more complicated

- We don’t know where points are :(
- We don’t know which go together :(
Things get more complicated

• We don’t know where points are :(
• We don’t know which go together :(
Things get more complicated

- We don’t know where points are :(
- We don’t know which go together :(
Things get more complicated

- We don’t know where points are :(  
- We don’t know which go together :( 

\[
\begin{bmatrix}
  x_1 & \cdot \\
  y_1 & y_2 \\
  \cdot & z_2
\end{bmatrix}
\]
Unions of Subspaces are Varieties!
Unions of Subspaces are Varieties!

\[ \mathcal{V} = \left\{ \mathbf{x} : \begin{align*} xy(x - y) & = 0 \\ xy(x - z) & = 0 \\ z(x - y) & = 0 \\ z(x - z) & = 0 \end{align*} \right\} \]
Unions of Subspaces are Varieties!

\[
\mathcal{V} = \left\{ \begin{align*}
xy(x - y) &= 0 \\
xy(x - z) &= 0 \\
z(x - y) &= 0 \\
z(x - z) &= 0
\end{align*} \right\}
\]

\[
= \left\{ \begin{align*}
-\frac{5}{6}xy + \frac{1}{2}xz + \frac{5}{30}yz + \frac{5}{36}z^2 &= 0 \\
\frac{5}{6}xy - \frac{1}{2}xz + \frac{5}{30}yz - \frac{5}{36}z^2 &= 0 \\
\frac{5}{30}xy - \frac{1}{2}xz - \frac{5}{30}yz + \frac{5}{36}z^2 &= 0
\end{align*} \right\}
\]
Unions of Subspaces are Varieties!

\[ V = \left\{ x : \begin{array}{l} xy(x - y) = 0 \\ xy(x - z) = 0 \\ z(x - y) = 0 \\ z(x - z) = 0 \end{array} \right\} \]

\[ = \left\{ x : \begin{array}{l} -5xy + \frac{1}{2}xz + \frac{5}{30}yz + \frac{5}{30}z^2 = 0 \\ -\frac{5}{30}xy - \frac{1}{2}xz + \frac{5}{6}yz - \frac{5}{30}z^2 = 0 \\ -\frac{5}{30}xy - \frac{1}{2}xz - \frac{5}{30}yz + \frac{5}{6}z^2 = 0 \end{array} \right\} \]

\[ = \left\{ x : x^{\otimes 2} \in \ker V^T =: S \right\} \]
Unions of Subspaces are Varieties!

So we can use our theory!
Information-theoretic requirements

Previously known under random samplings

Eriksson et. al, 2012

Samples per column

\( \ell \)

\( \Theta(r \log d) \)

0

\( \Theta(d^{\log d}) \) Columns

\( N_k \)
Information-theoretic requirements

\[ \ell = \frac{r(d - r)}{N_k} + r \]

Previous known under random samplings

Samples per column

\[ \mathcal{O}(r \log d) \]
\[ \mathcal{O}(\max\{r, \log d\}) \]
\[ r + 1 \]

Impossible

Possible, if entries are observed in the right places

Possible, under random samplings

Eriksson et. al, 2012

\[ \mathcal{O}(d^{\log d}) \] Columns

Theory matches Practice

Impossible
Possible, if entries are observed in the right places
Previously known under random samplings
Possible, under random samplings

\[ t = \frac{r(d - r)}{N_k} + r \]

Samples per Column

Number of Columns

GSSC  MSC  EM
Why do you want to know?

Mmmh... You begin to convince me...
Why do you want to know?

Mmmh... You begin to convince me...

(Tough crowd!)
Why do you want to know? Mmmh… You begin to convince me…

(Tough crowd!)

All right, Here are other Applications…
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery
Drug Discovery

Columns in Subspace?
Drug Discovery
Drug Discovery
Drug Discovery

Adaptive Sampling

Columns in Subspace? Union? Variety?
Recommender Systems

- Non-uniform Sampling!
- Coherent Subspace?
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!
Countless Applications

- Non-uniform Sampling
- Coherent Subspace
Countless Applications

- Non-uniform Sampling
- Coherent Subspace
WOW, AMAZING

PLEASE TELL ME MORE
WOW, AMAZING
PLEASE TELL ME MORE

How am I on time?
WOW, AMAZING
PLEASE TELL ME MORE

How am I on time?

Beyond Missing Data
LRMC  
(Low-Rank Matrix Completion)

LR Matrix

---

RPCA  
(Robust Principal Component Analysis)

LR Matrix
LRMC
(Low-Rank Matrix Completion)

Tons of Missing Entries

LR Matrix

RPCA
(Robust Principal Component Analysis)
LRMC
(Low-Rank Matrix Completion)

LR Matrix

Tons of Missing Entries

RPCA
(Robust Principal Component Analysis)

LR Matrix

LR Matrix

Few Gross Errors
LRMC
(Low-Rank Matrix Completion)

Know Locations
☑ Don’t know values

Few Gross Errors

RPCA
(Robust Principal Component Analysis)

Tons of Missing Entries
**LRMC**  
(Low-Rank Matrix Completion)

- LR Matrix
- Tons of Missing Entries

- Know Locations
- Don’t know values

**RPCA**  
(Robust Principal Component Analysis)

- LR Matrix
- Few Gross Errors

- Don’t know Locations
- Know all values
LRMC
(Low-Rank Matrix Completion)

LR Matrix
Tons of Missing Entries

Know Locations
Don’t know values

RPCA
(Robust Principal Component Analysis)

LR Matrix
Few Gross Errors

Don’t know Locations
Know all values

Common goal: find the Subspace
Background segmentation
Background segmentation
Background segmentation
Existing Approaches

\[
\begin{align*}
\text{minimize} & \quad \|L\|_* + \lambda \|S\|_1 \\
\text{subject to} & \quad X = L + S
\end{align*}
\]


Existing Approaches

\[
\begin{align*}
\text{minimize} & \quad \|\mathbf{L}\|_* + \lambda \|\mathbf{S}\|_1 \\
\text{subject to} & \quad \mathbf{X} = \mathbf{L} + \mathbf{S}
\end{align*}
\]

Using our Theory
Totally different way to think about the problem

Use incomplete-data tricks
Use incomplete-data tricks

rank=$r$

uncorrupted
Use incomplete-data tricks

corrupted
Use incomplete-data tricks
Use incomplete-data tricks

Keep finding uncorrupted pieces
Use incomplete-data tricks

Keep finding uncorrupted pieces
Use incomplete-data tricks

Keep finding uncorrupted pieces
Use incomplete-data tricks

Keep finding uncorrupted pieces
Use incomplete-data tricks

Each piece gives you a Projection!
Use incomplete-data tricks

If pieces are observed in the right places, we can find the subspace

Each piece gives you a Projection!
Use incomplete-data tricks

If pieces are observed in the right places, we can find the subspace efficiently.
Background segmentation
In many cases, similar results
In other cases, better
In other cases, better
Performance Analysis
Performance Analysis

Few errors

Coherent (bad)

Incoherent (good)
Performance Analysis

- Coherent (bad)
- Incoherent (good)

Few errors vs. Many errors

\[ \mu \] vs. \( p \)
Performance Analysis

Coherent (bad)

Incoherent (good)

Few errors

Many errors

RPCA-ALM (Lin et al., 2011-2016)

The lighter the better
Performance Analysis

Coherent (bad)
Incoherent (good)

Few errors
Many errors


(the lighter the better)
Original Video

Our Work

RPCA-ALM
RPCA-ALM (Lin et al., 2011-2016)
To Sum up...
To Sum up...
To Sum up...
To Sum up…
To Sum up...
To Sum up…
Thank you
pimentel@gsu.edu

[1] Low Algebraic Dimension Matrix Completion


[4] The Information-Theoretic Requirements of Subspace Clustering with Missing Data

[5] Random Consensus Robust PCA

https://danielpimentel.github.io/publications