Selective Erasures for High-Dimensional Robust Subspace Tracking

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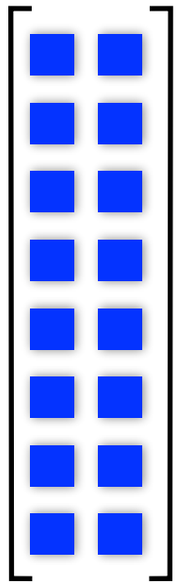
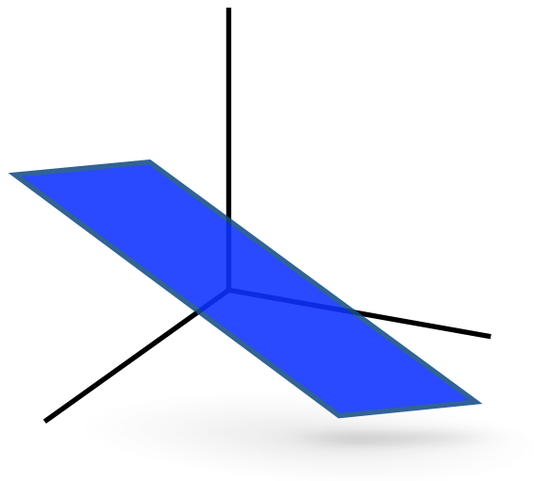
Computer Science

SPIE 2018

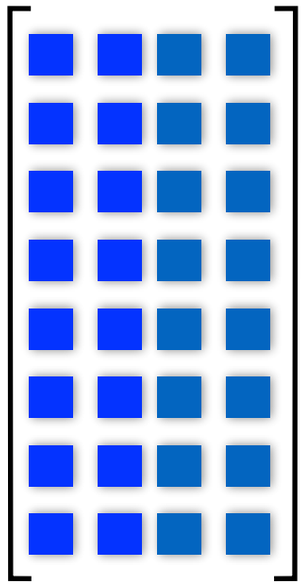


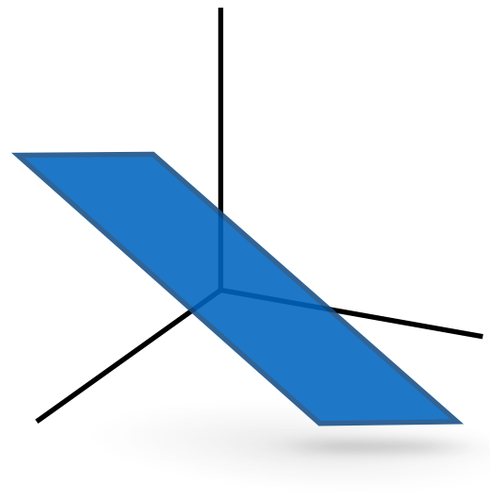


L

Low-Rank

Subspace Tracking





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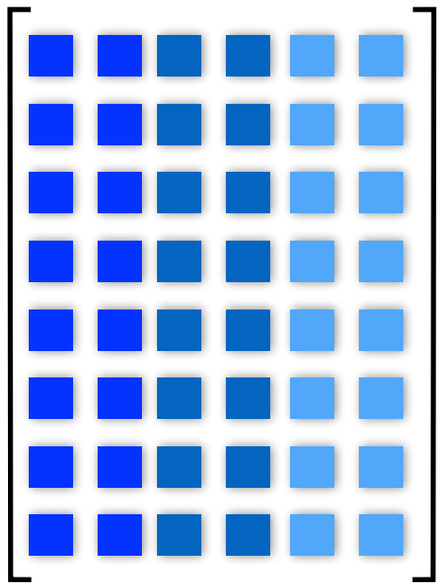

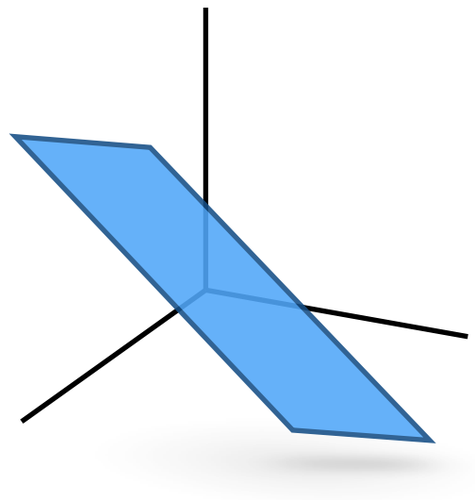

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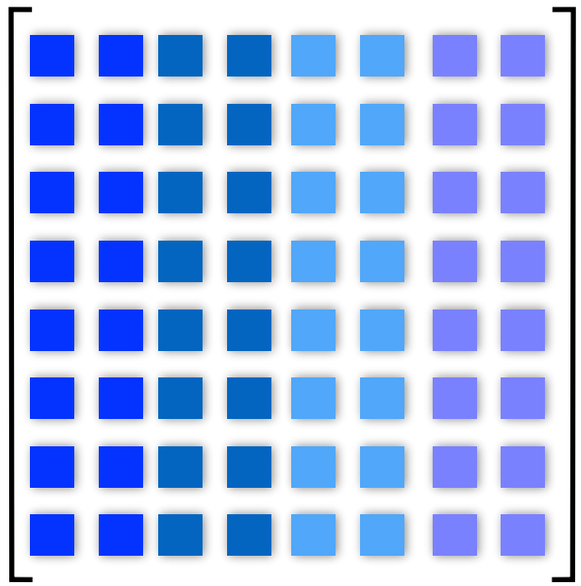
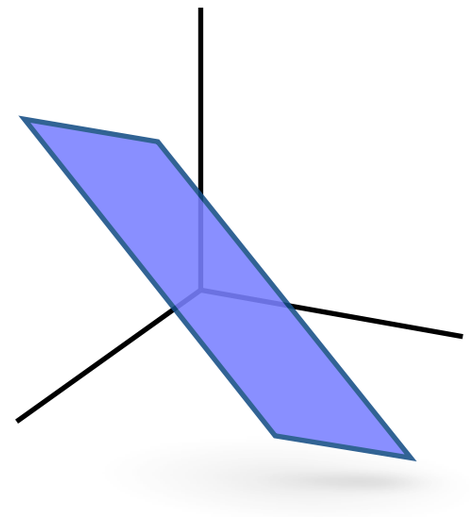
Subspace Tracking





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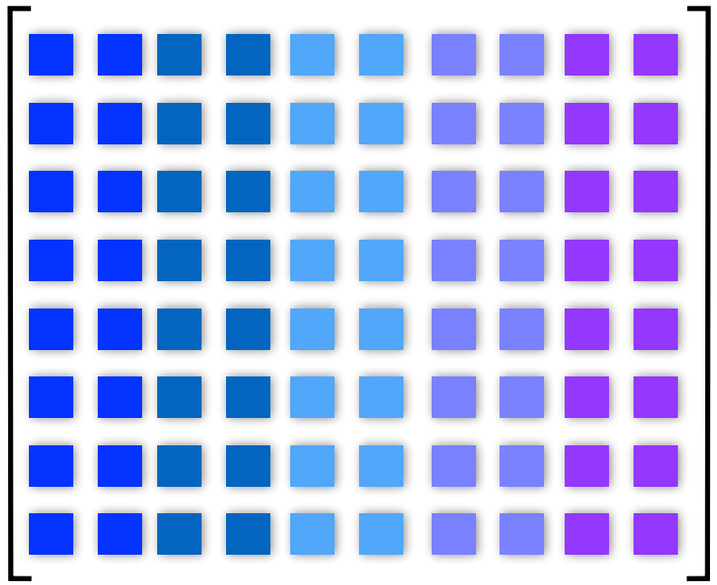
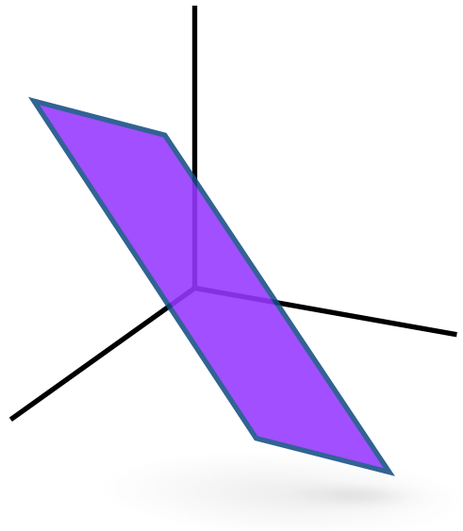
Subspace Tracking





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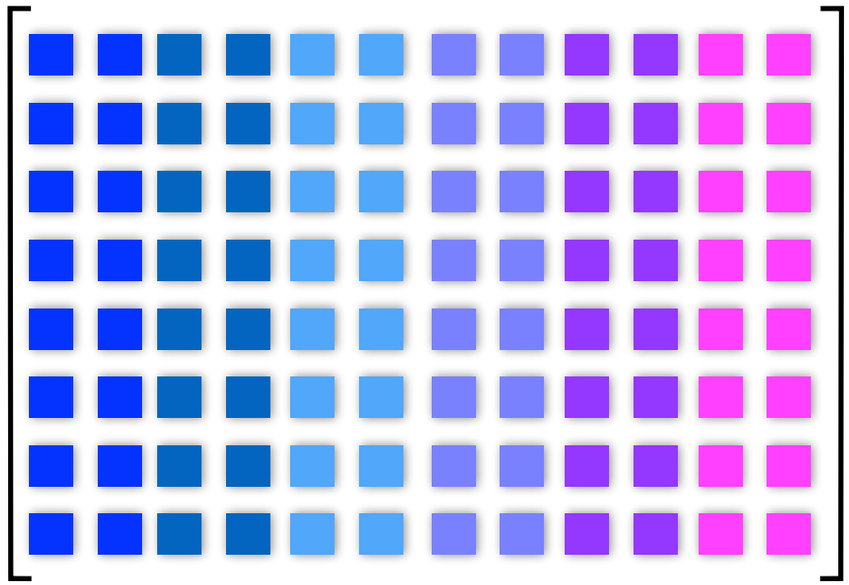
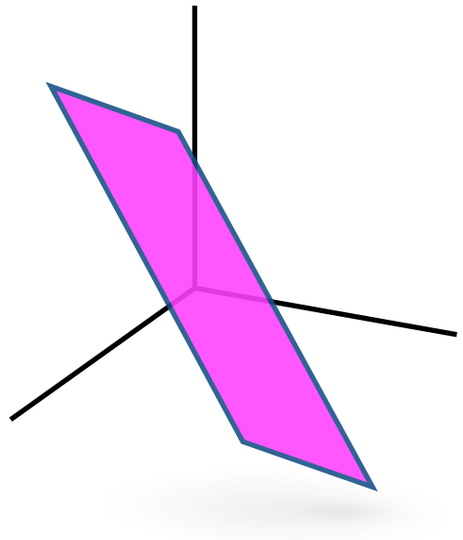
Subspace Tracking





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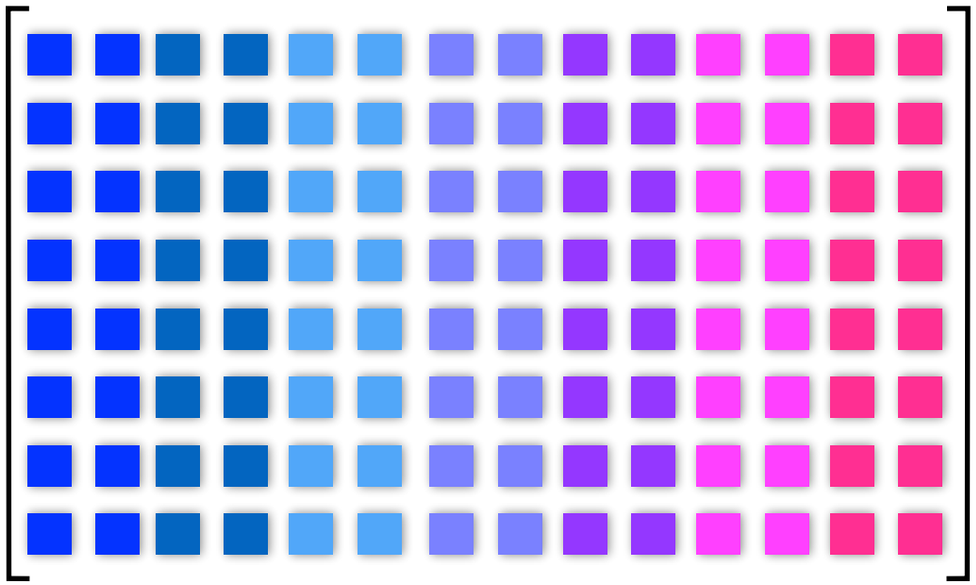
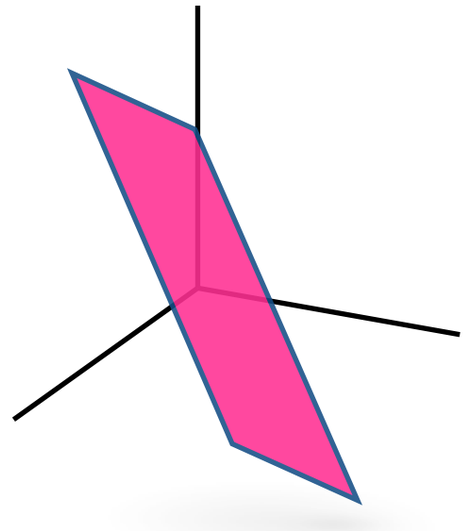
Subspace Tracking





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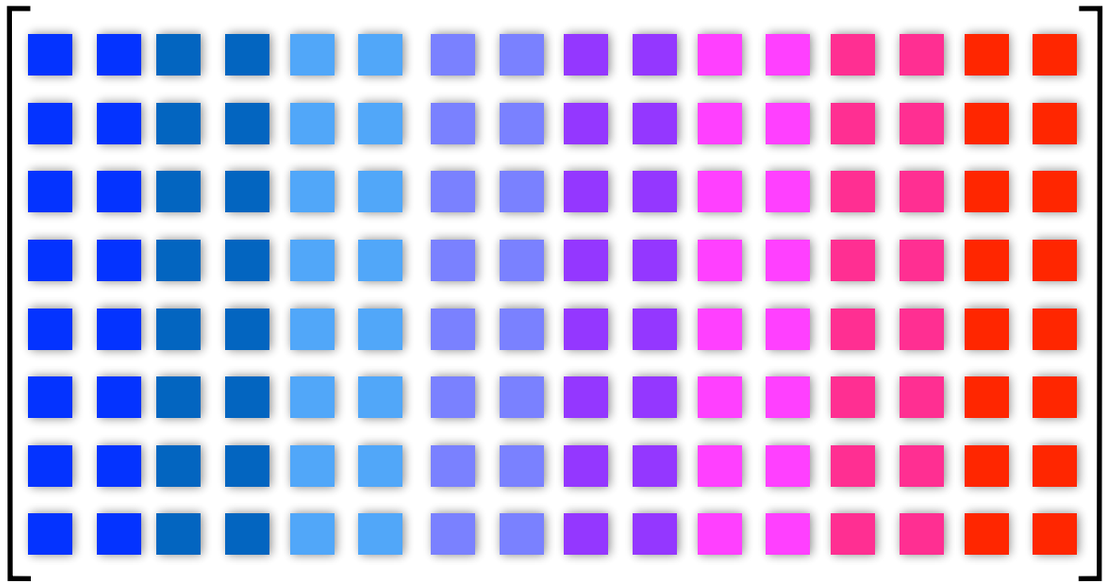
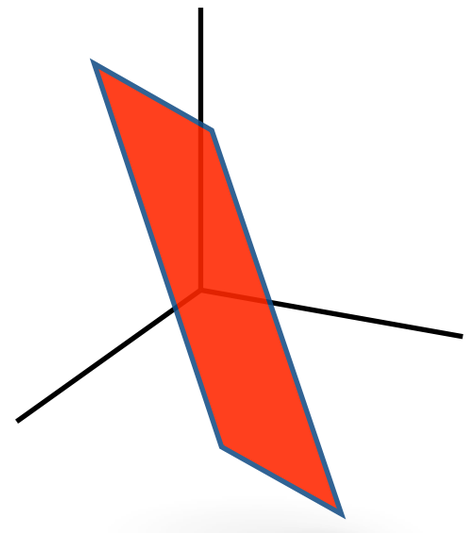
Subspace Tracking





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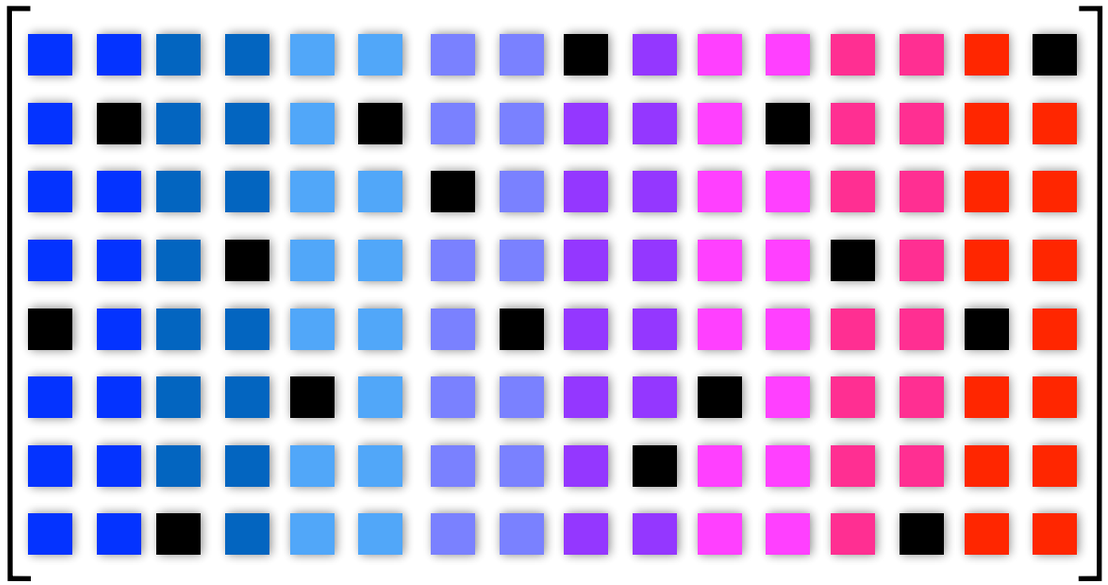
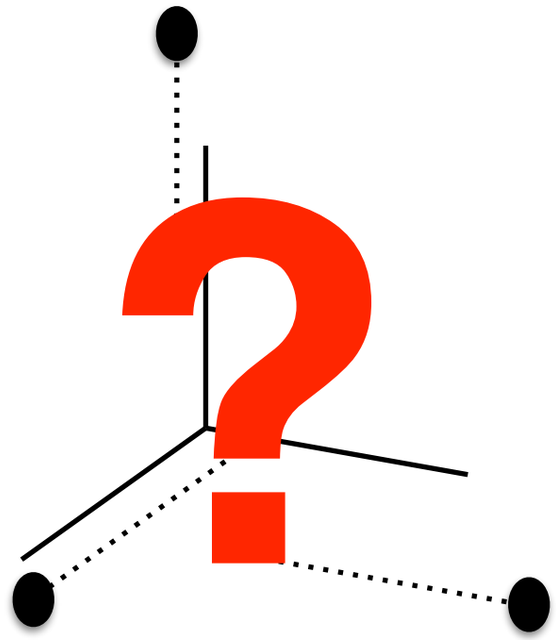


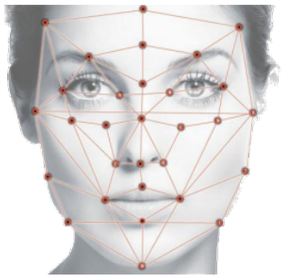


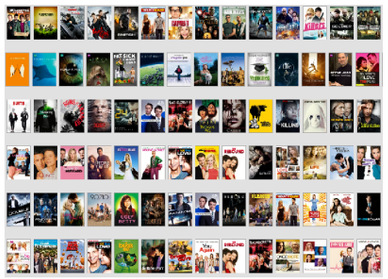
L + S

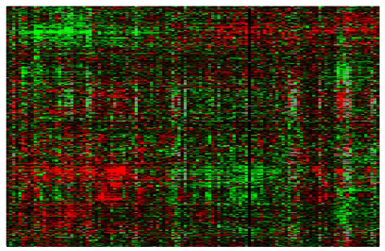
Low-Rank Sparse

Robust Subspace Tracking

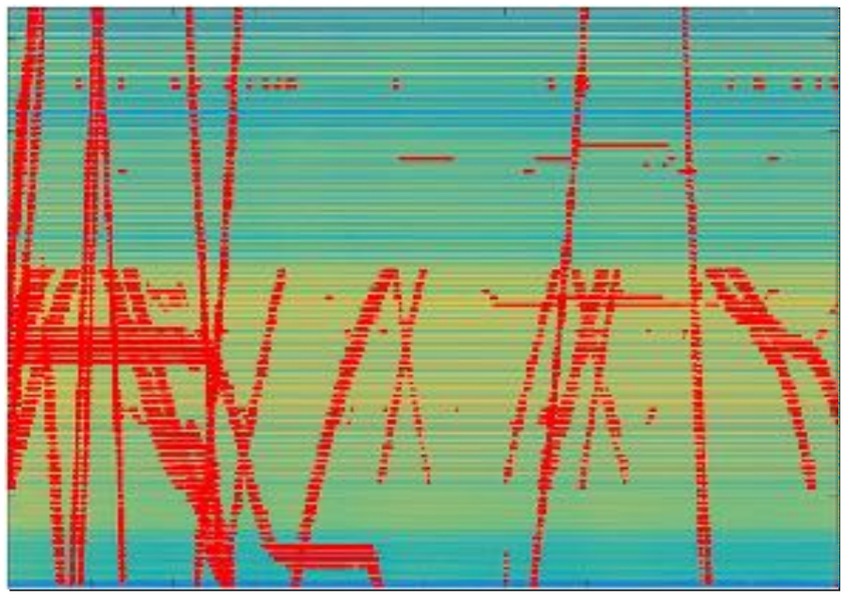

What is this good for?



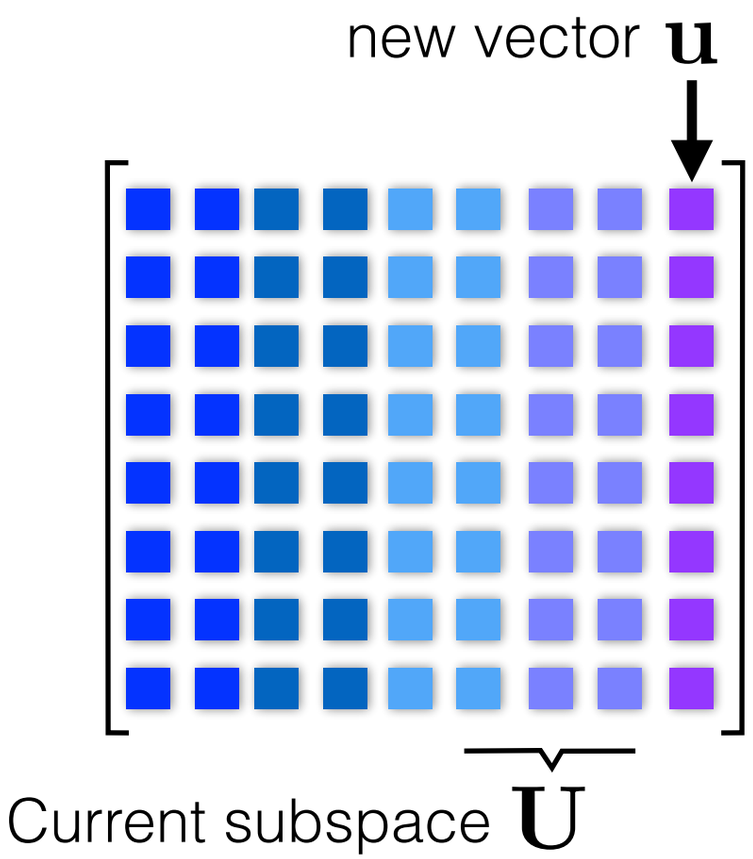


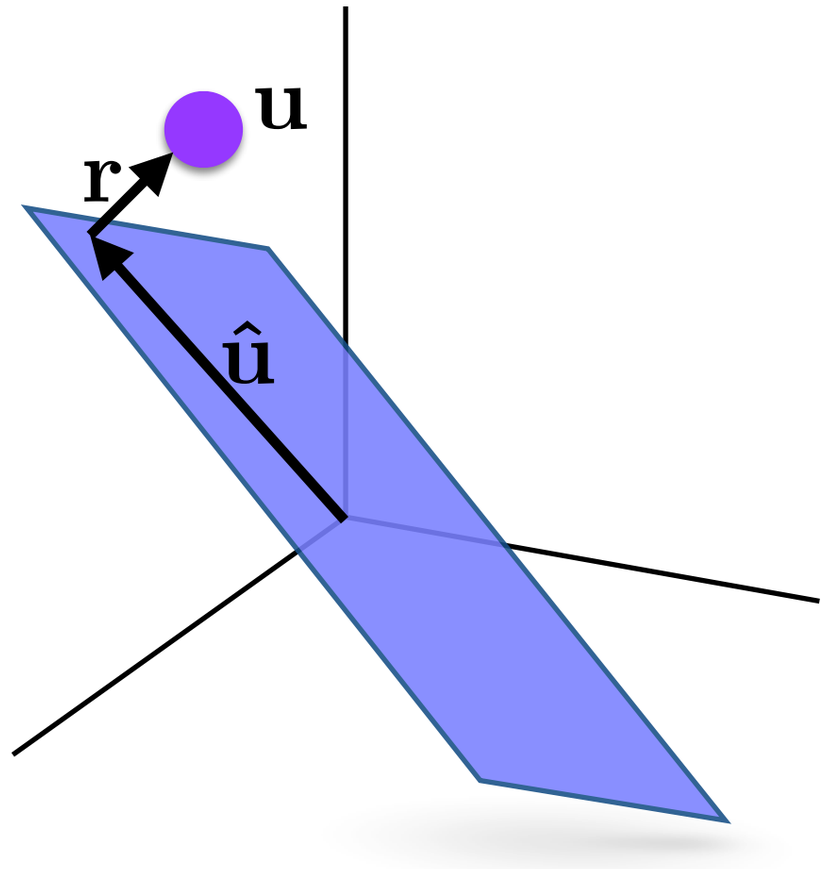


Lots of Applications



Background segmentation

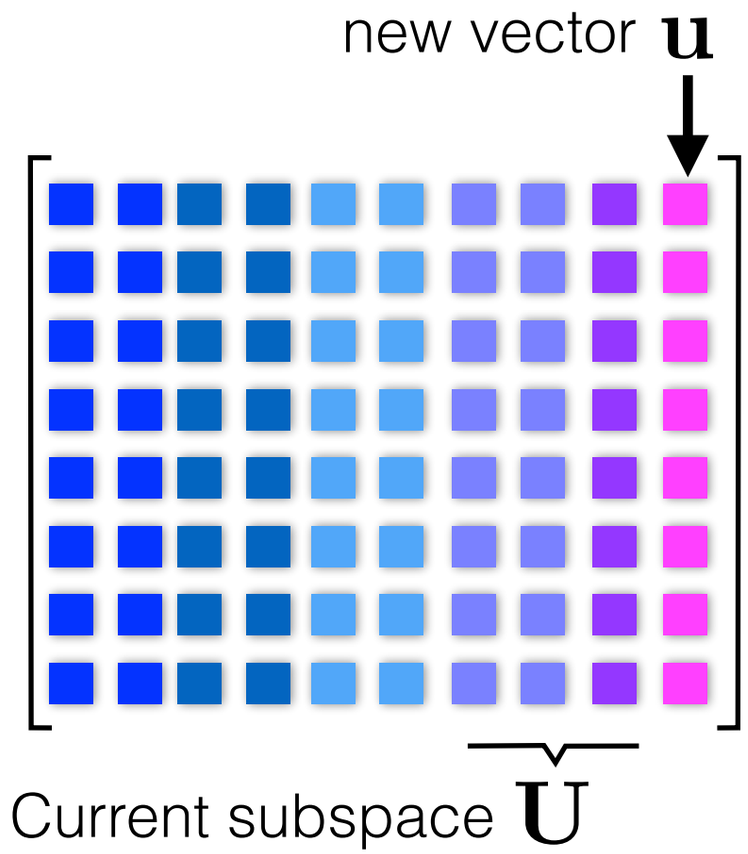


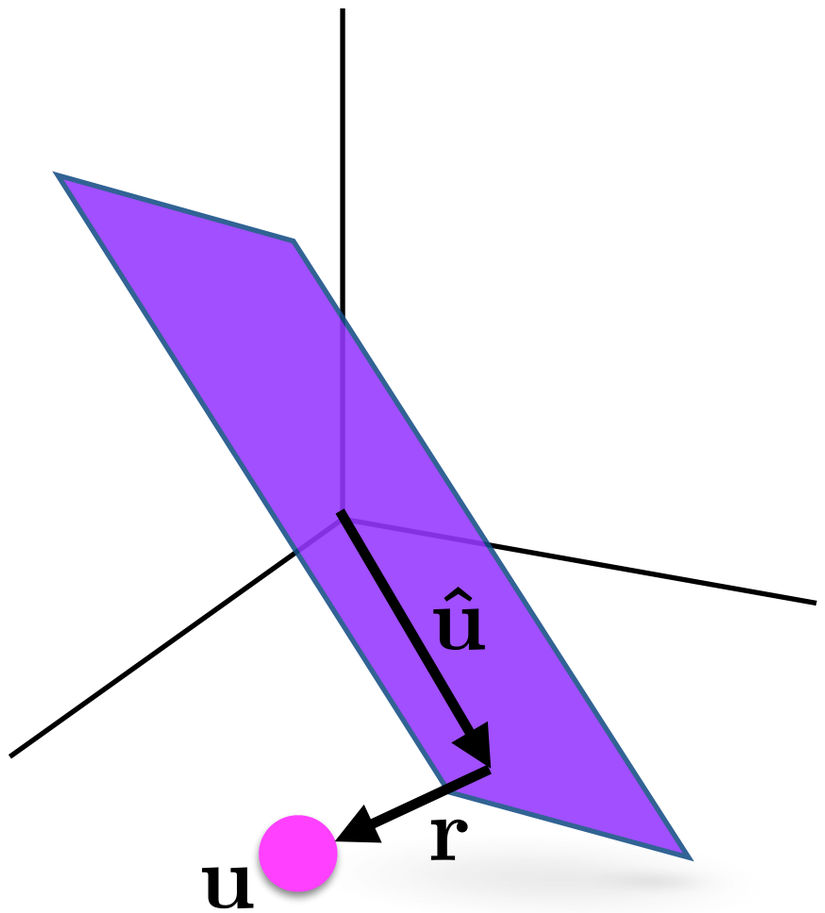


$$\text{Update} = \mathbf{U} + \left((\cos(\sigma\eta) - 1) \frac{\hat{\mathbf{u}}}{\|\hat{\mathbf{u}}\|} + \sin(\sigma\eta) \frac{\mathbf{r}}{\|\mathbf{r}\|} \right) \frac{\boldsymbol{\theta}^{\mathsf{T}}}{\|\boldsymbol{\theta}\|}$$

Existing theory

Forget about errors for a second…





$$\text{Update} = \mathbf{U} + \left((\cos(\sigma\eta) - 1) \frac{\hat{\mathbf{u}}}{\|\hat{\mathbf{u}}\|} + \sin(\sigma\eta) \frac{\mathbf{r}}{\|\mathbf{r}\|} \right) \frac{\boldsymbol{\theta}^{\mathsf{T}}}{\|\boldsymbol{\theta}\|}$$

Existing theory

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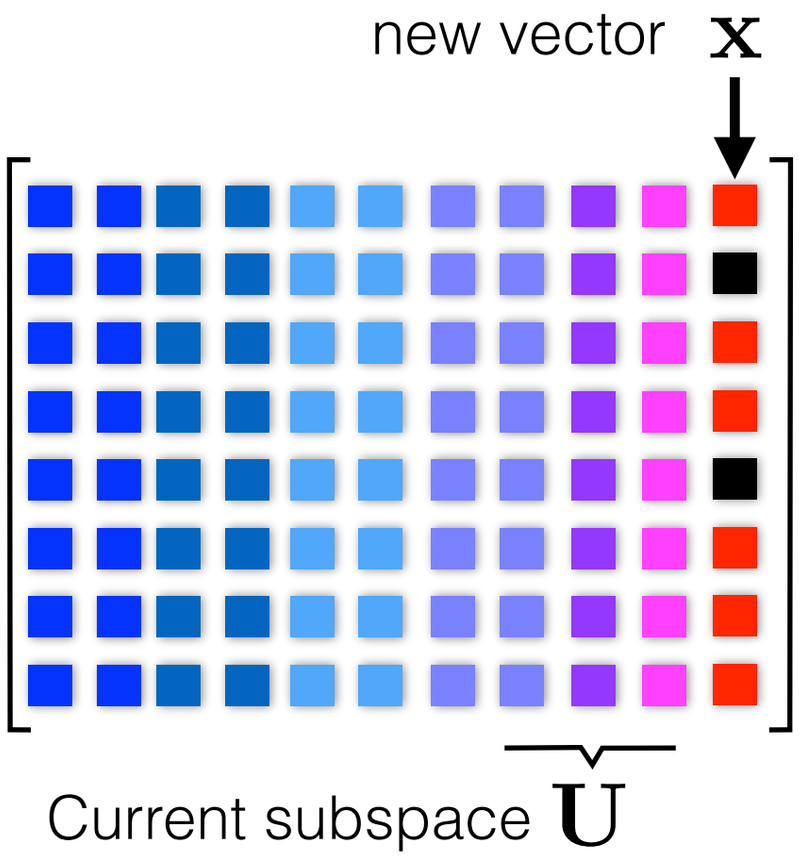
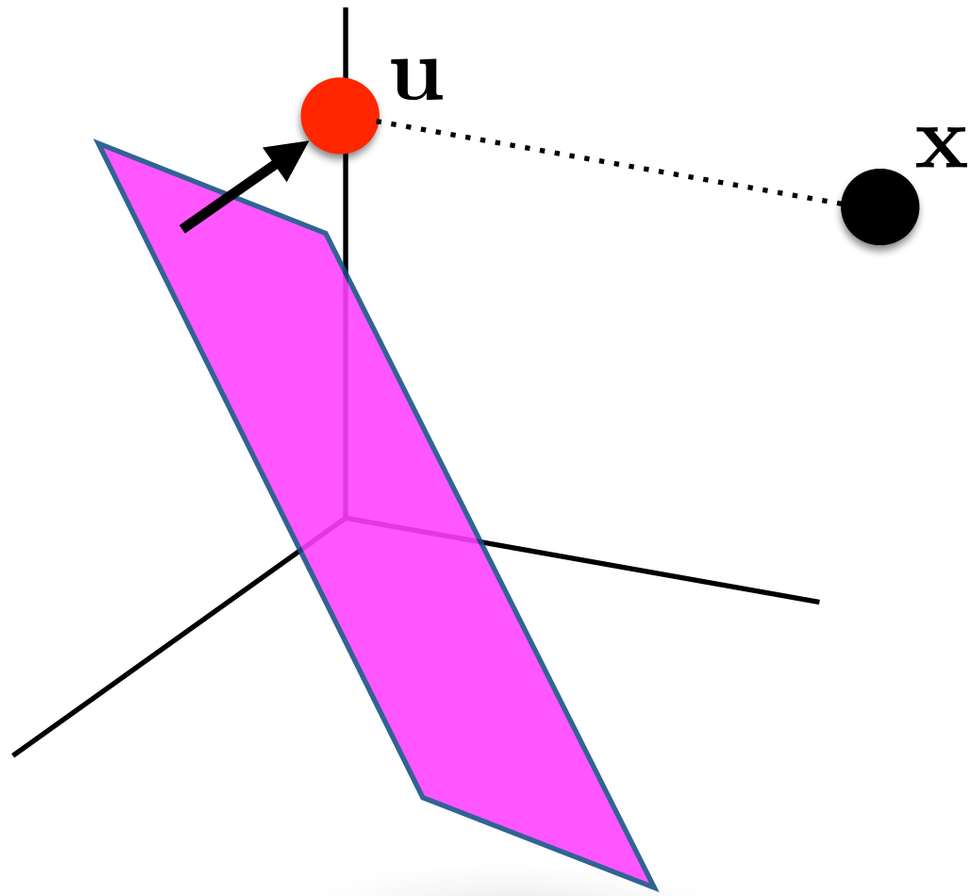
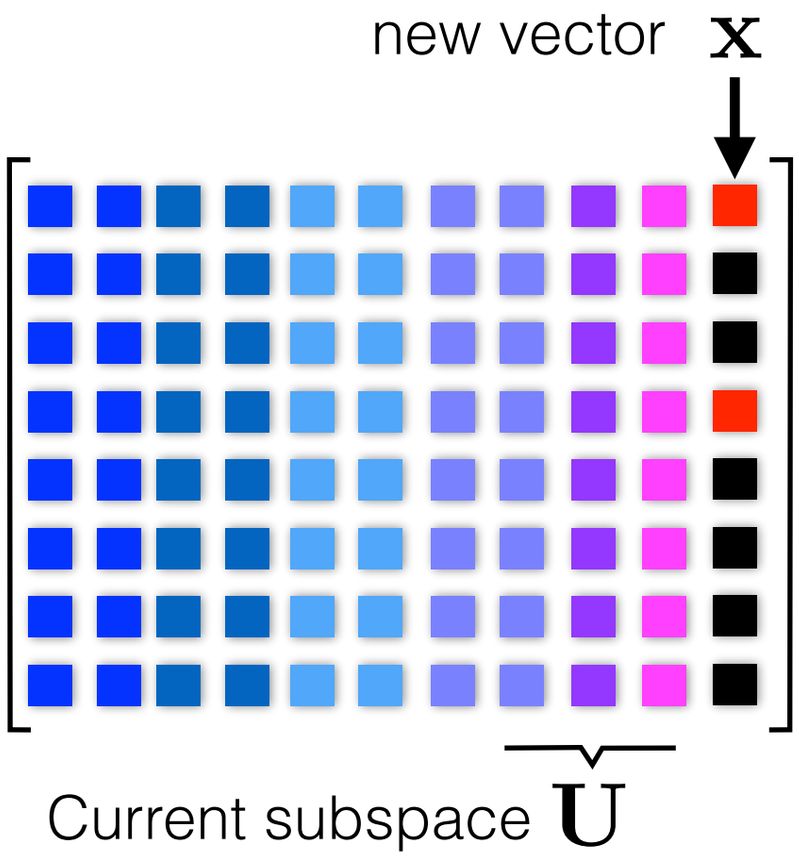


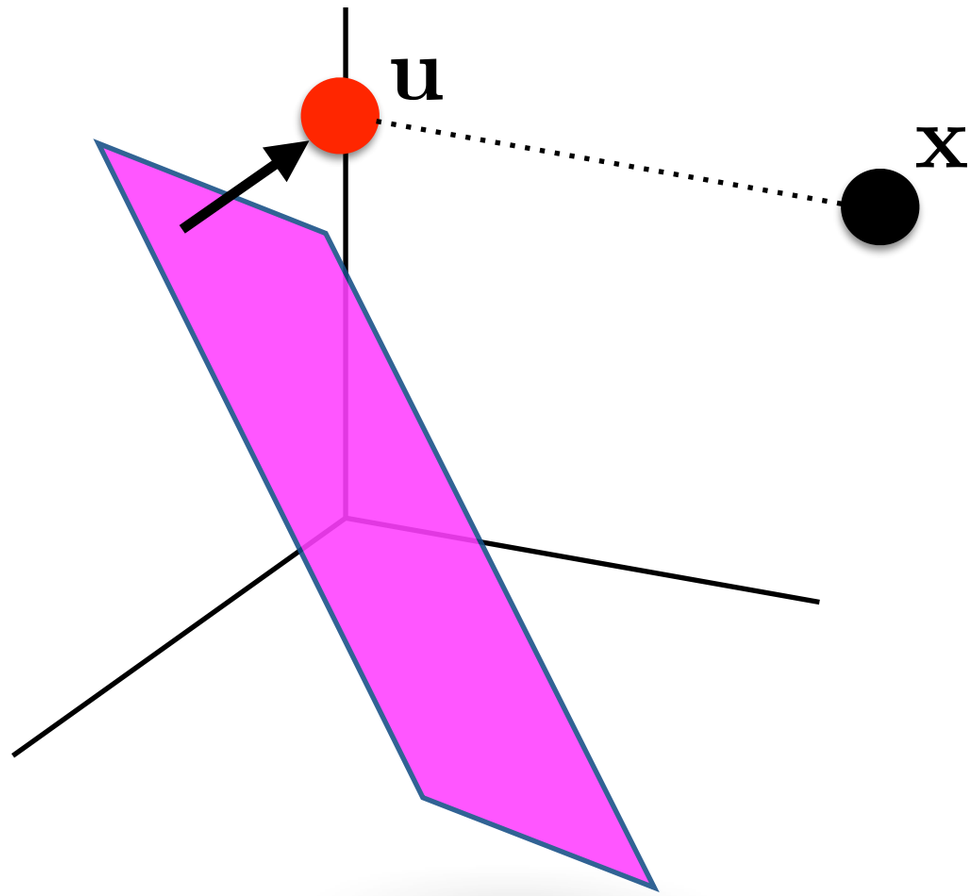


$$\min_{\boldsymbol{\theta} \in \mathbb{R}^{\mathbf{r}}} \| \overbrace{\mathbf{U}\boldsymbol{\theta}}^{\mathbf{u}} - \mathbf{x} \|_1$$ ← Solution with few nonzero entries

Existing theory

Now with sparse errors…





$$\min_{\boldsymbol{\theta} \in \mathbb{R}^r} \| \overbrace{\mathbf{U}\boldsymbol{\theta}}^{\mathbf{u}} - \mathbf{x} \|_1$$ ← Solution with few nonzero entries

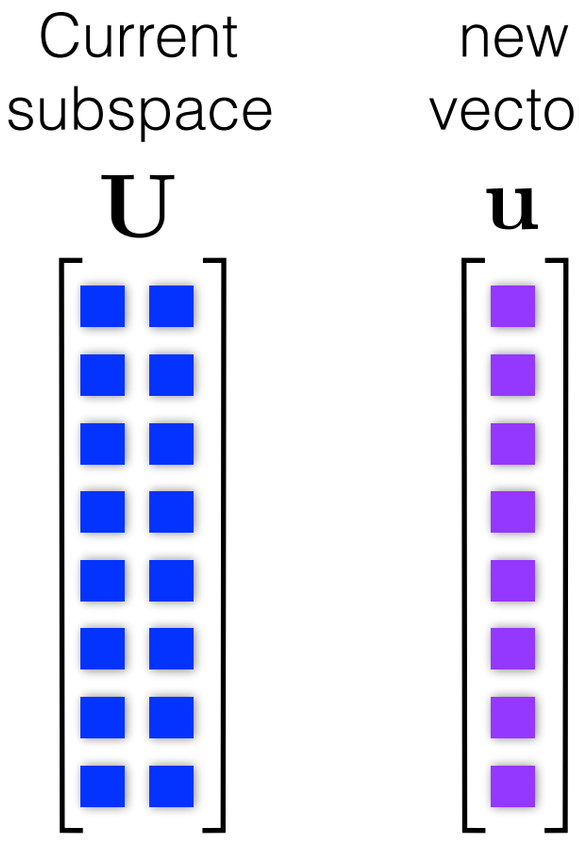
Won't work!!

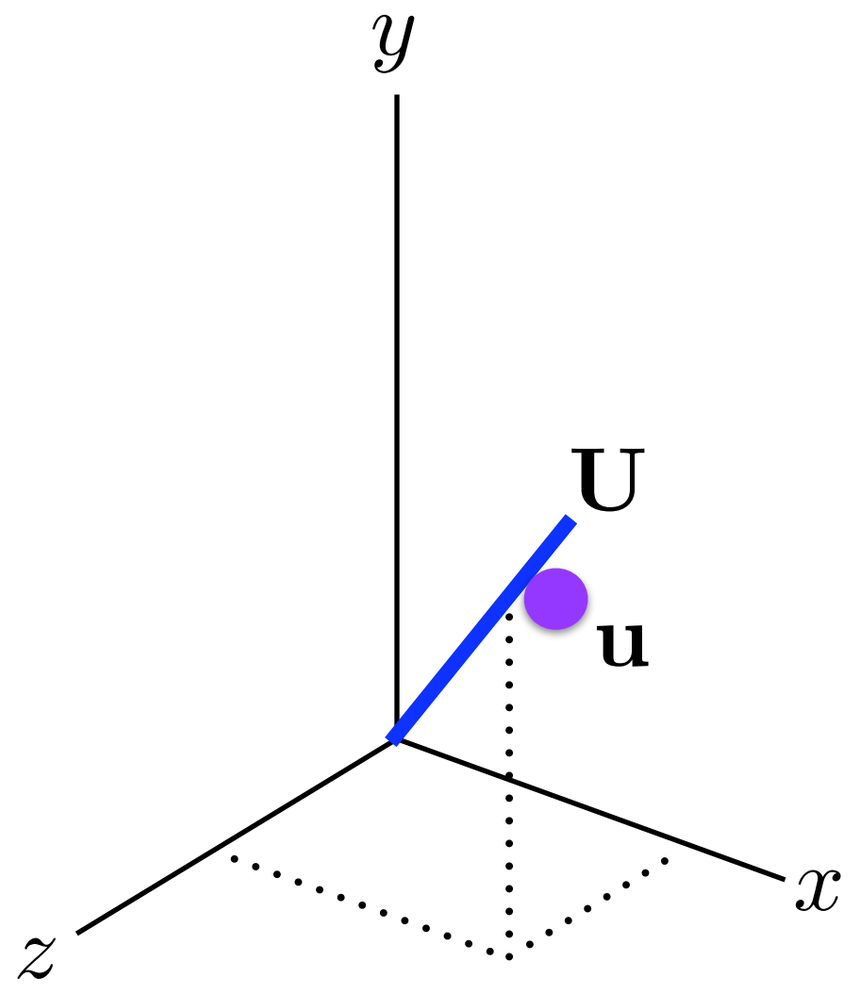
Existing theory

Too many errors?

Selective Erasures

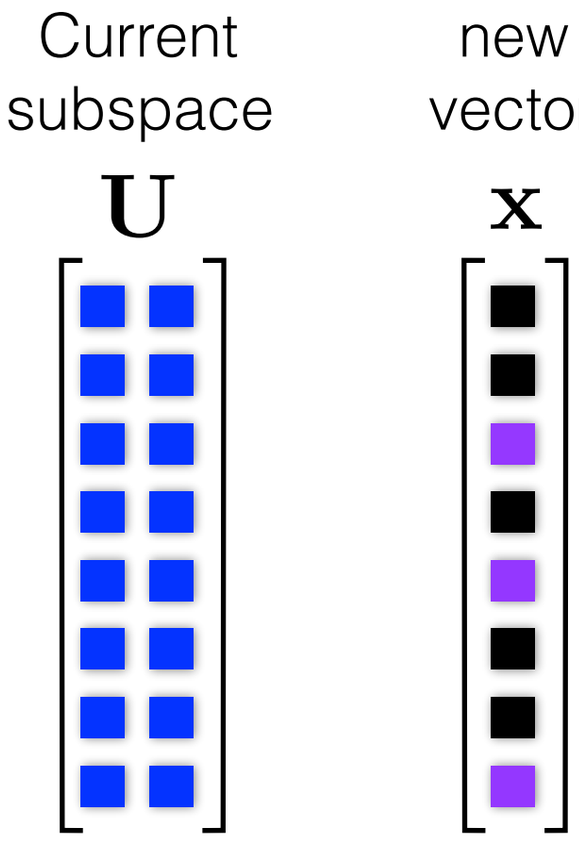
Our solution

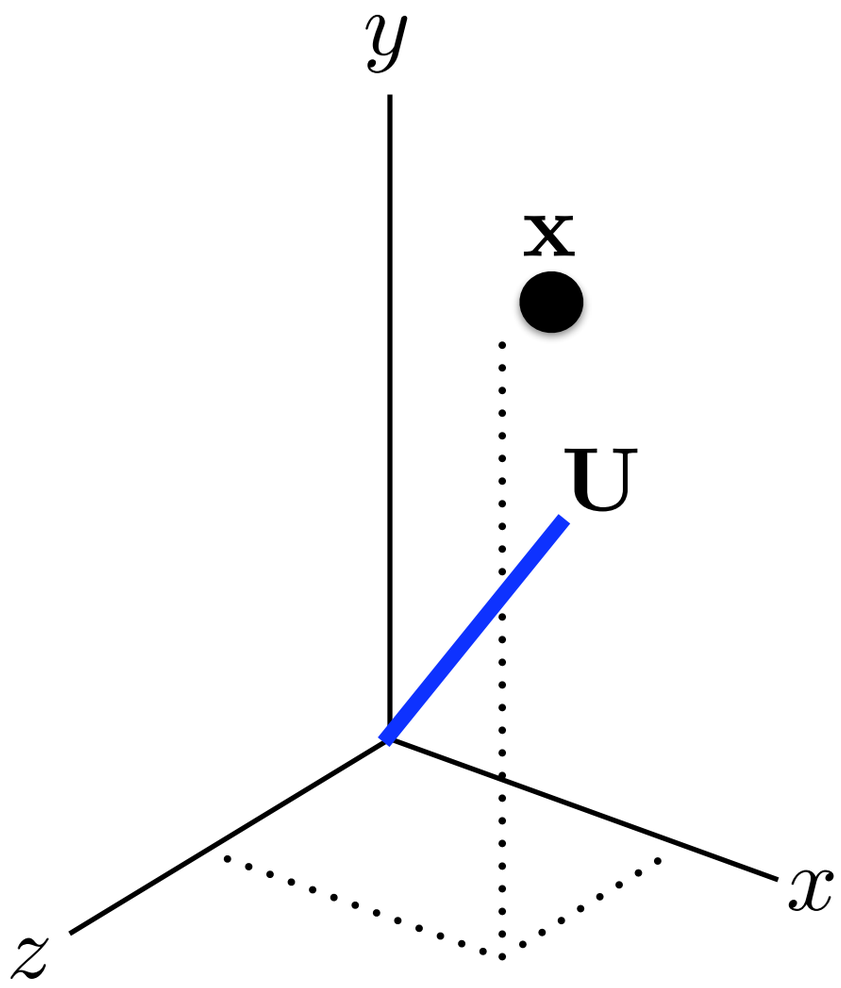




Main Idea

Remove coordinates that don’t help





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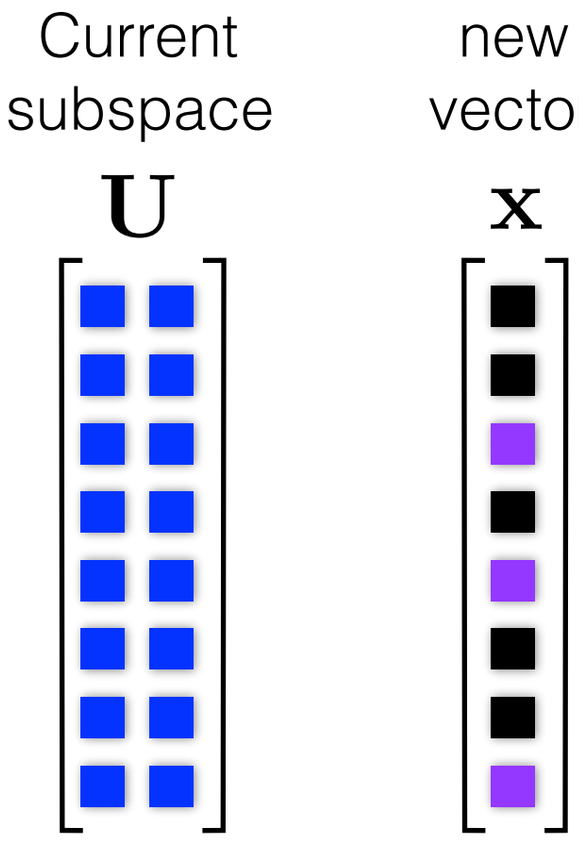

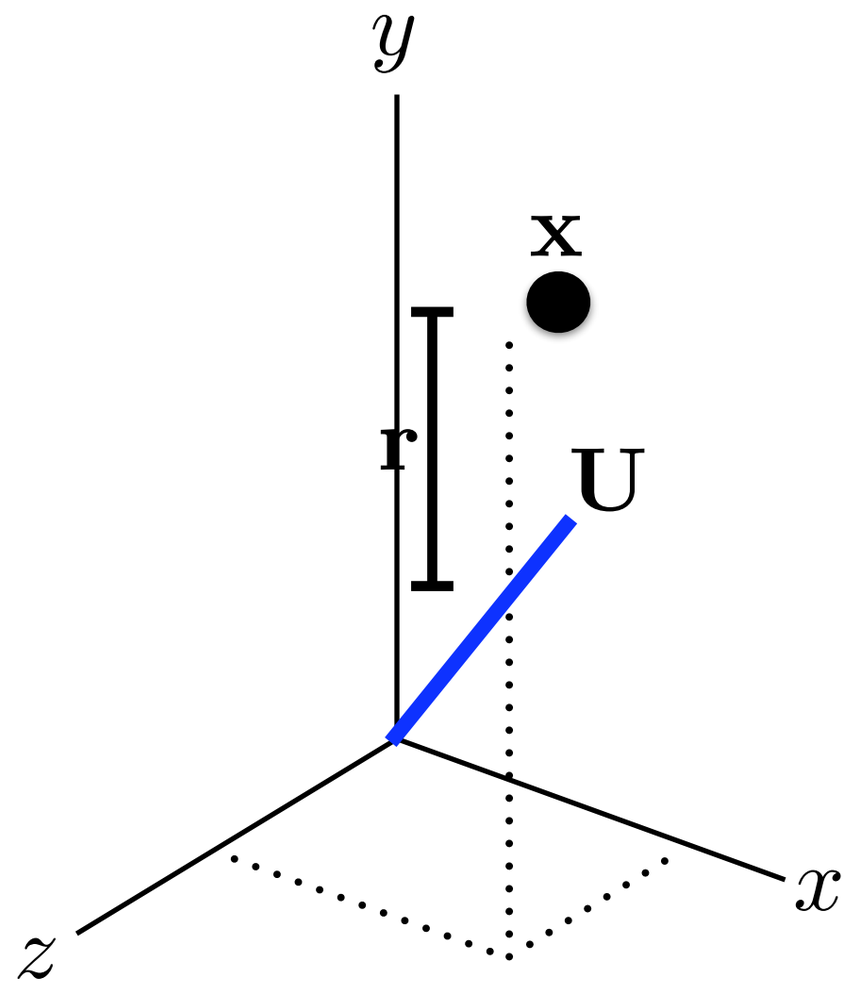

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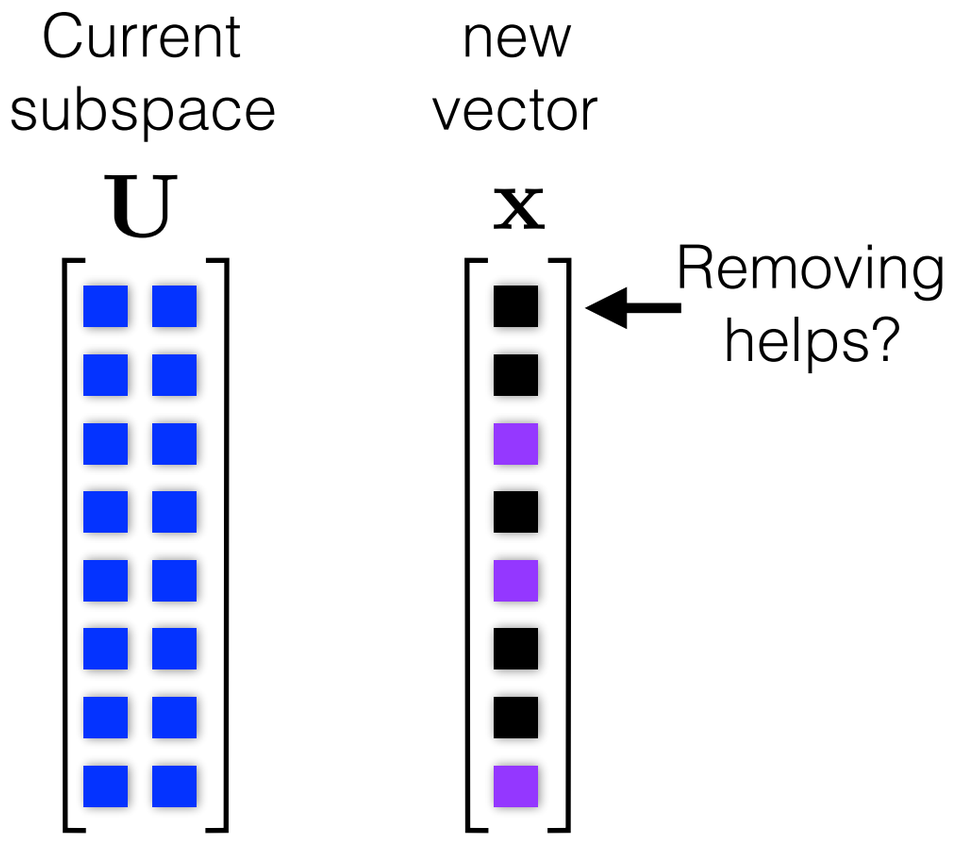

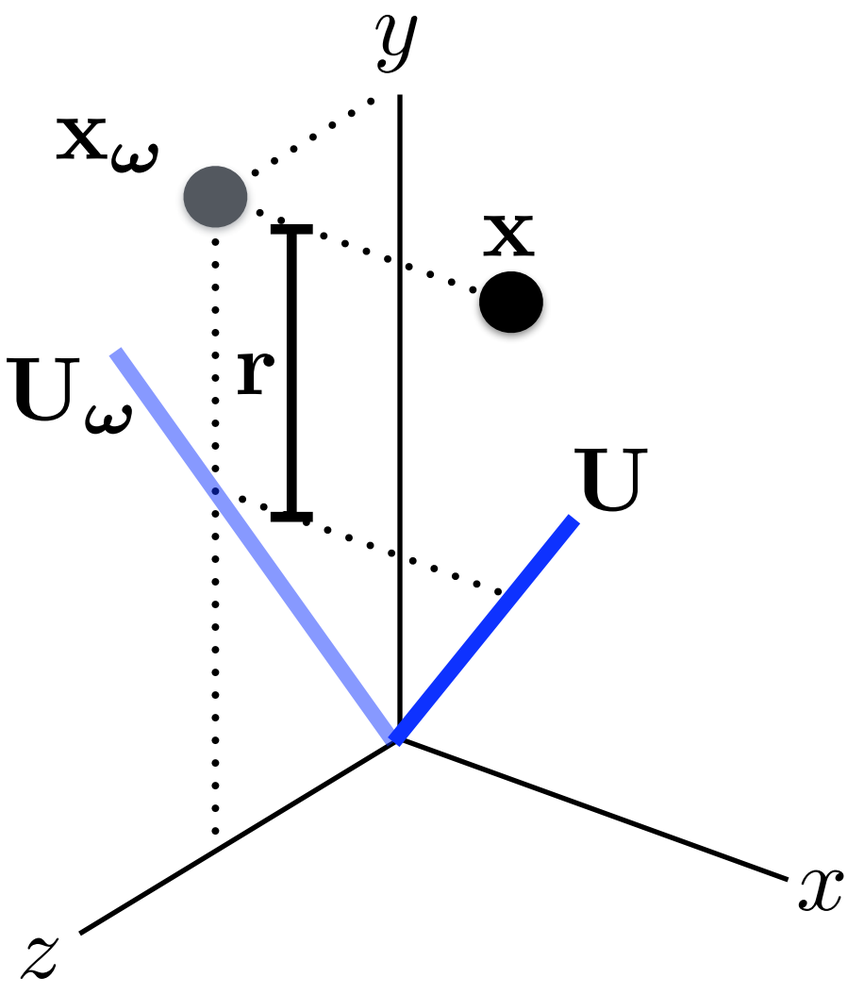

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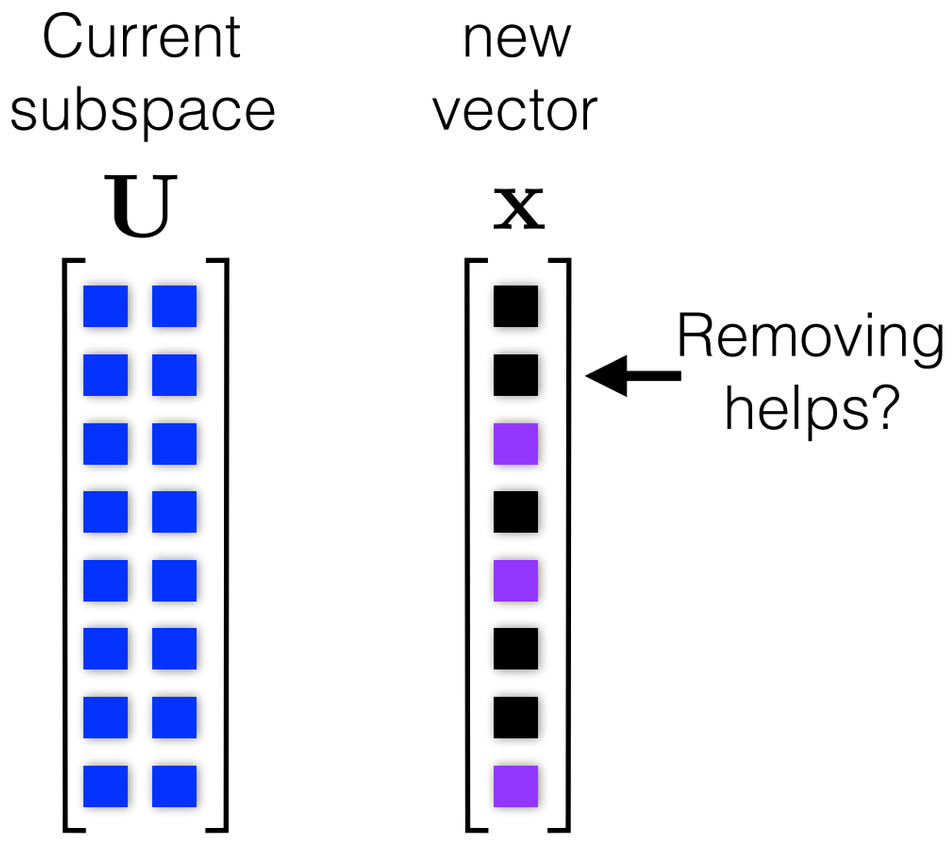

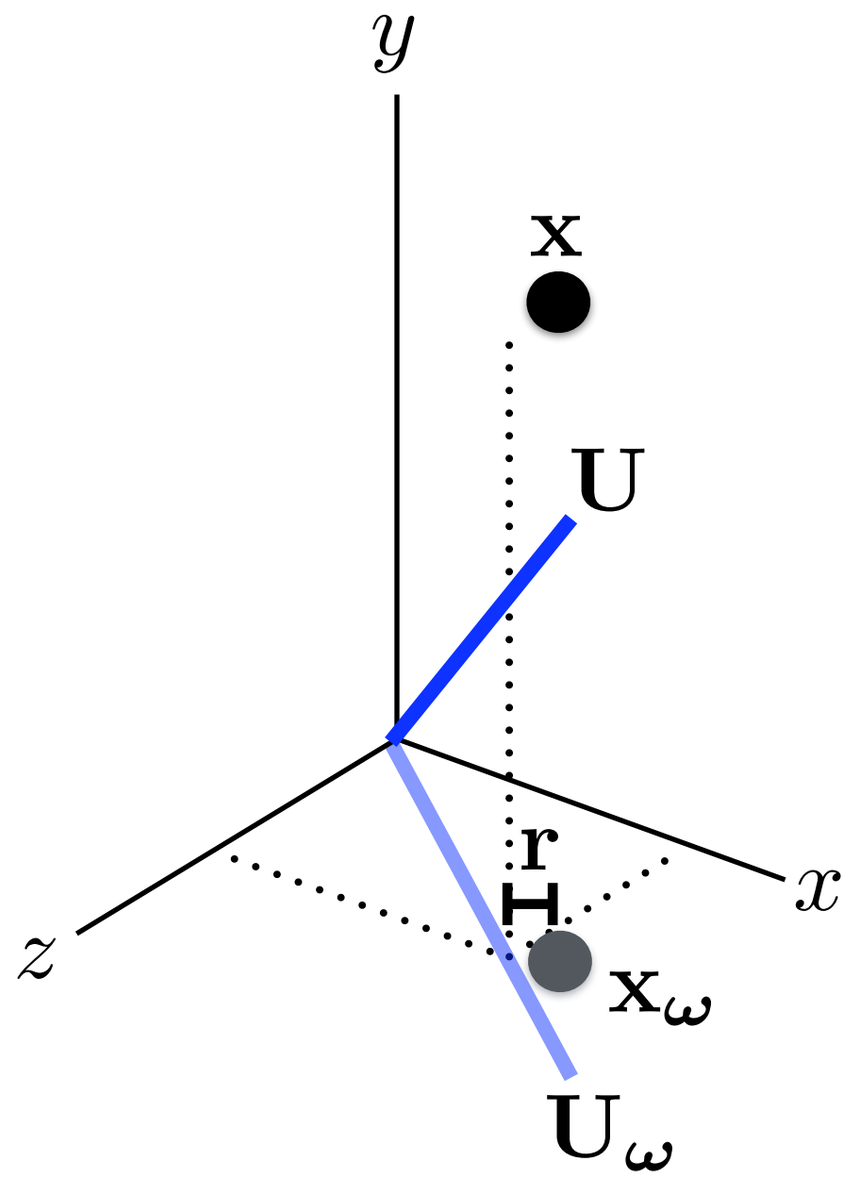

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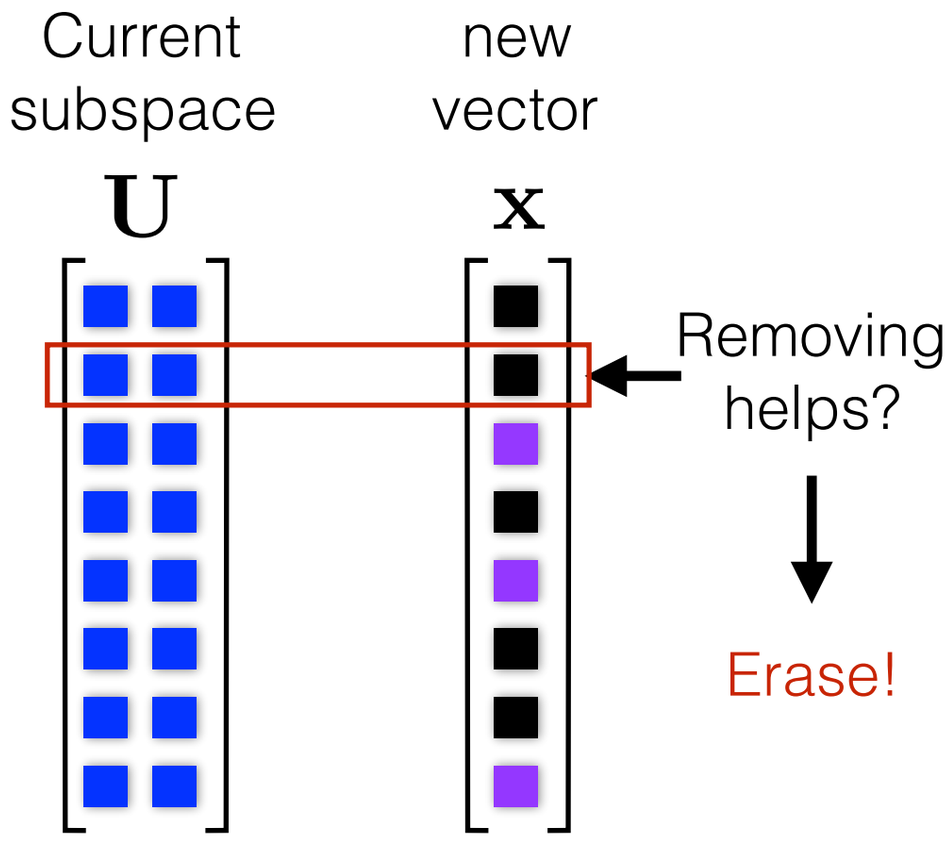

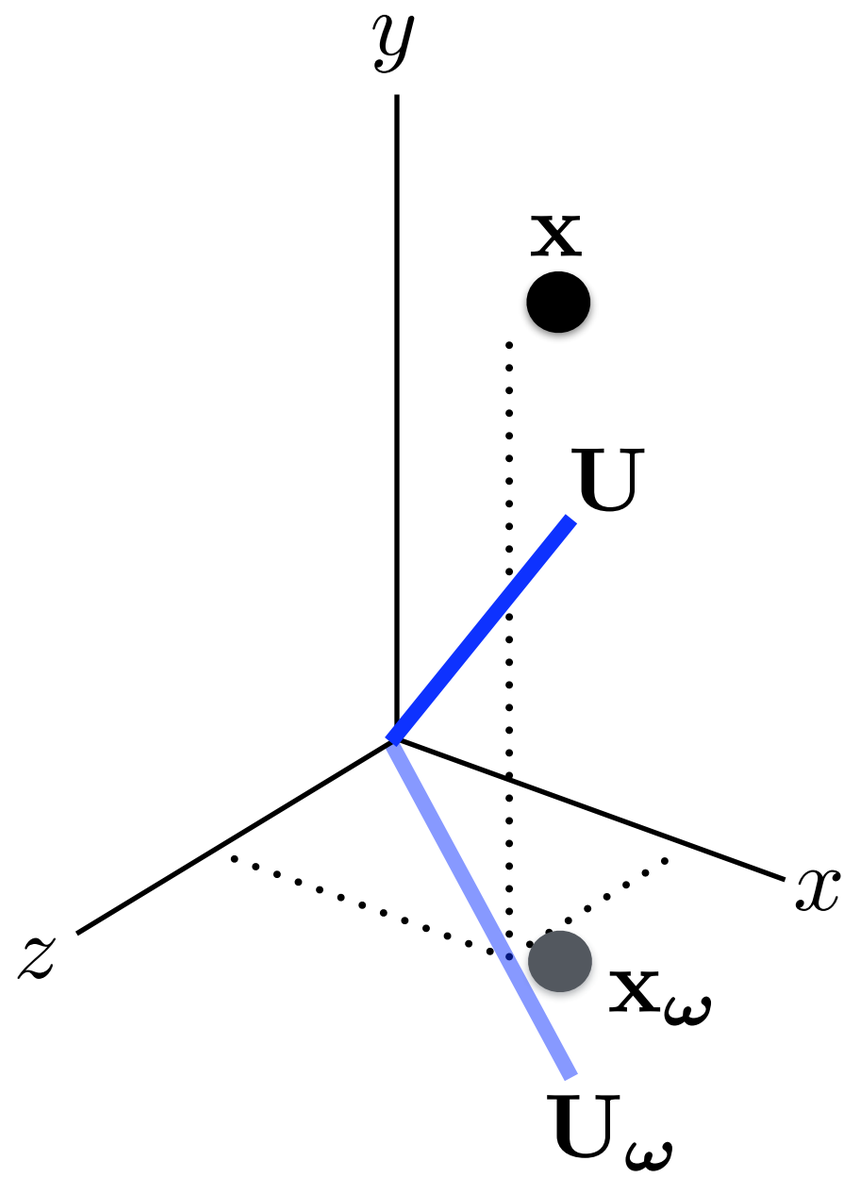

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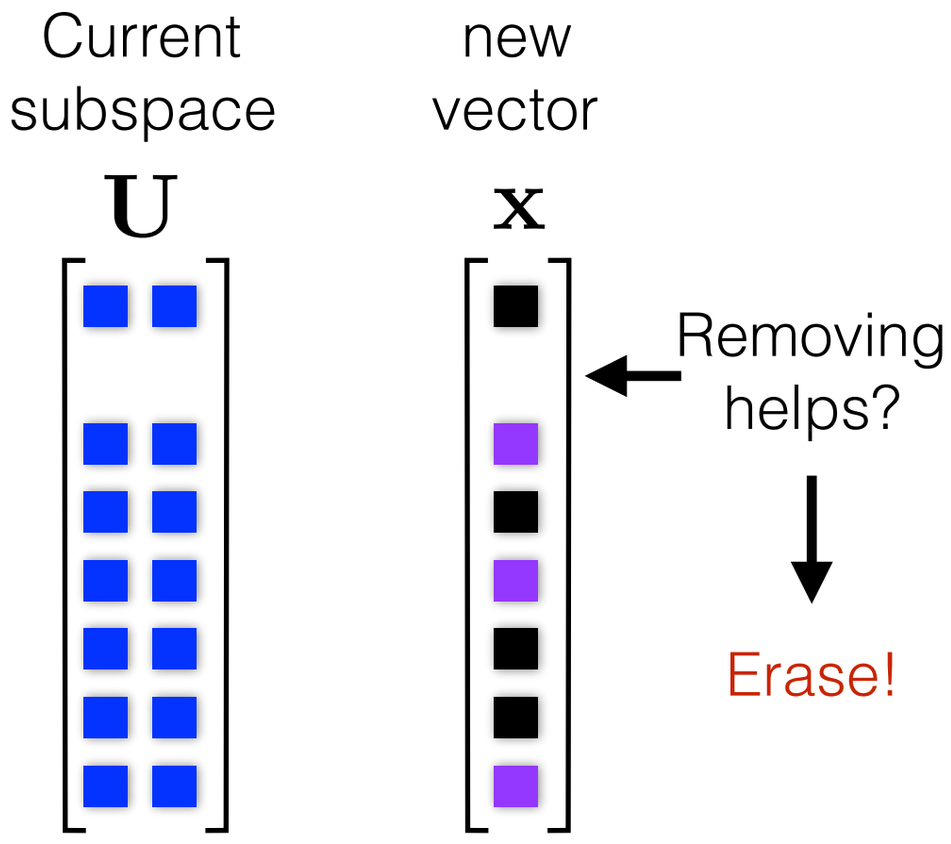

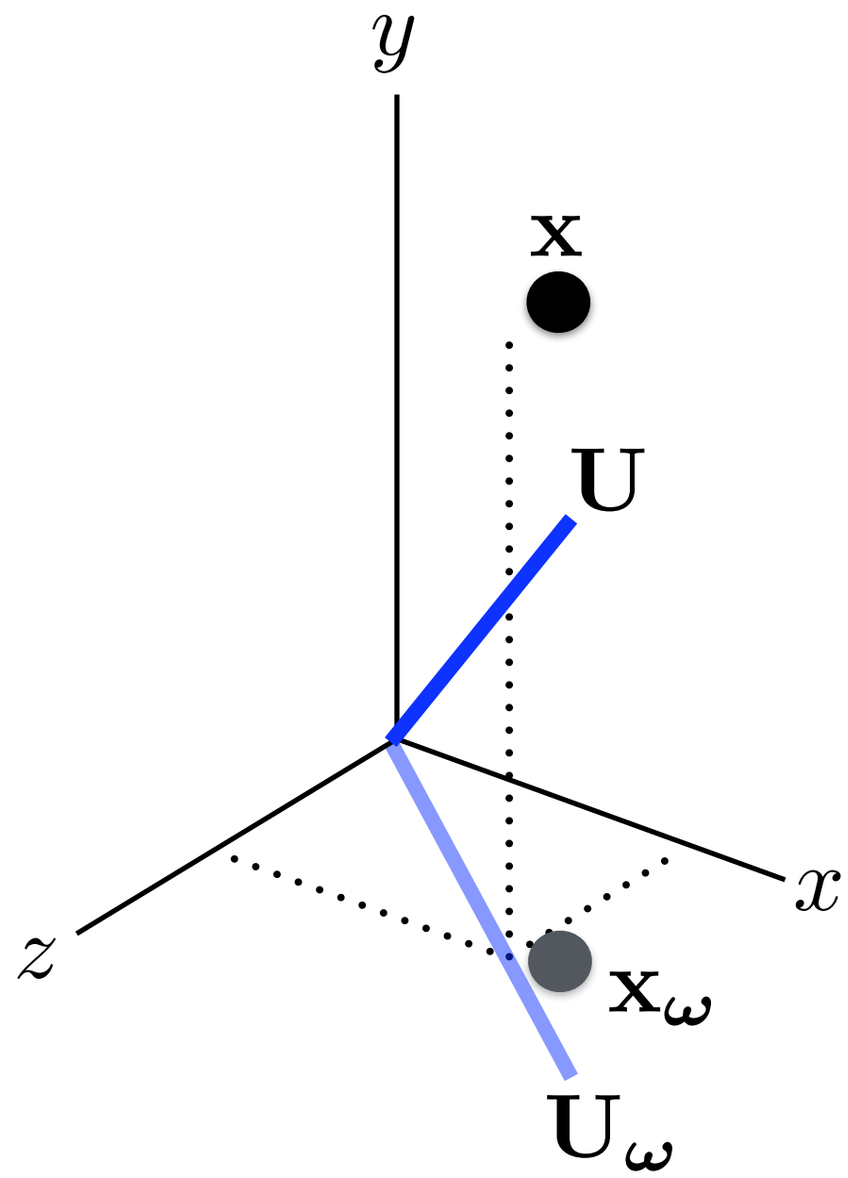

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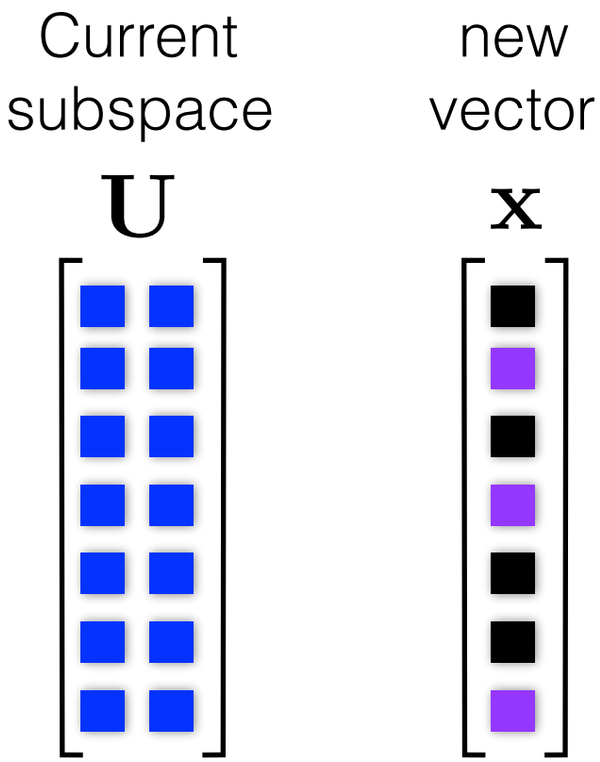
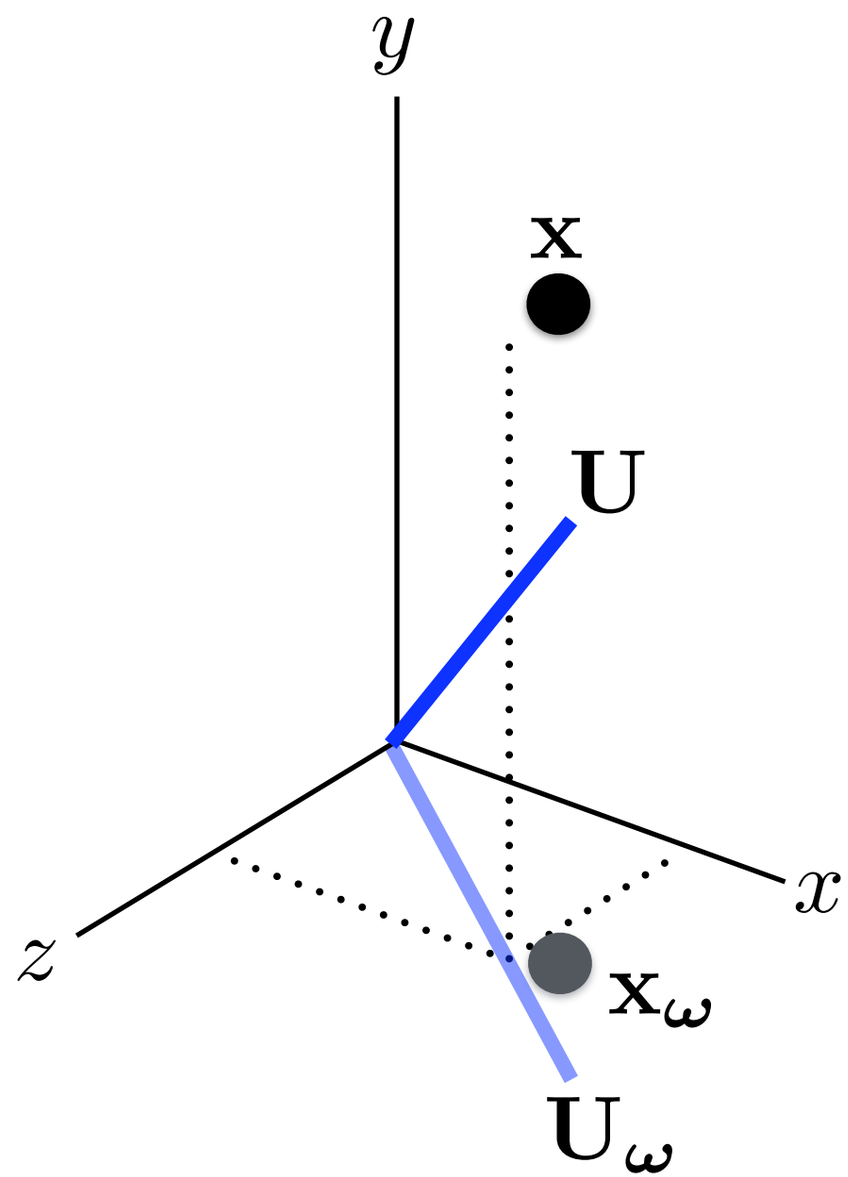


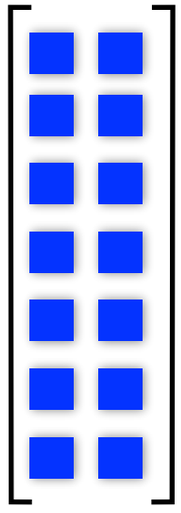


Main Idea

Remove coordinates that don’t help

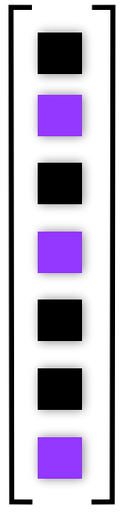
Current subspace

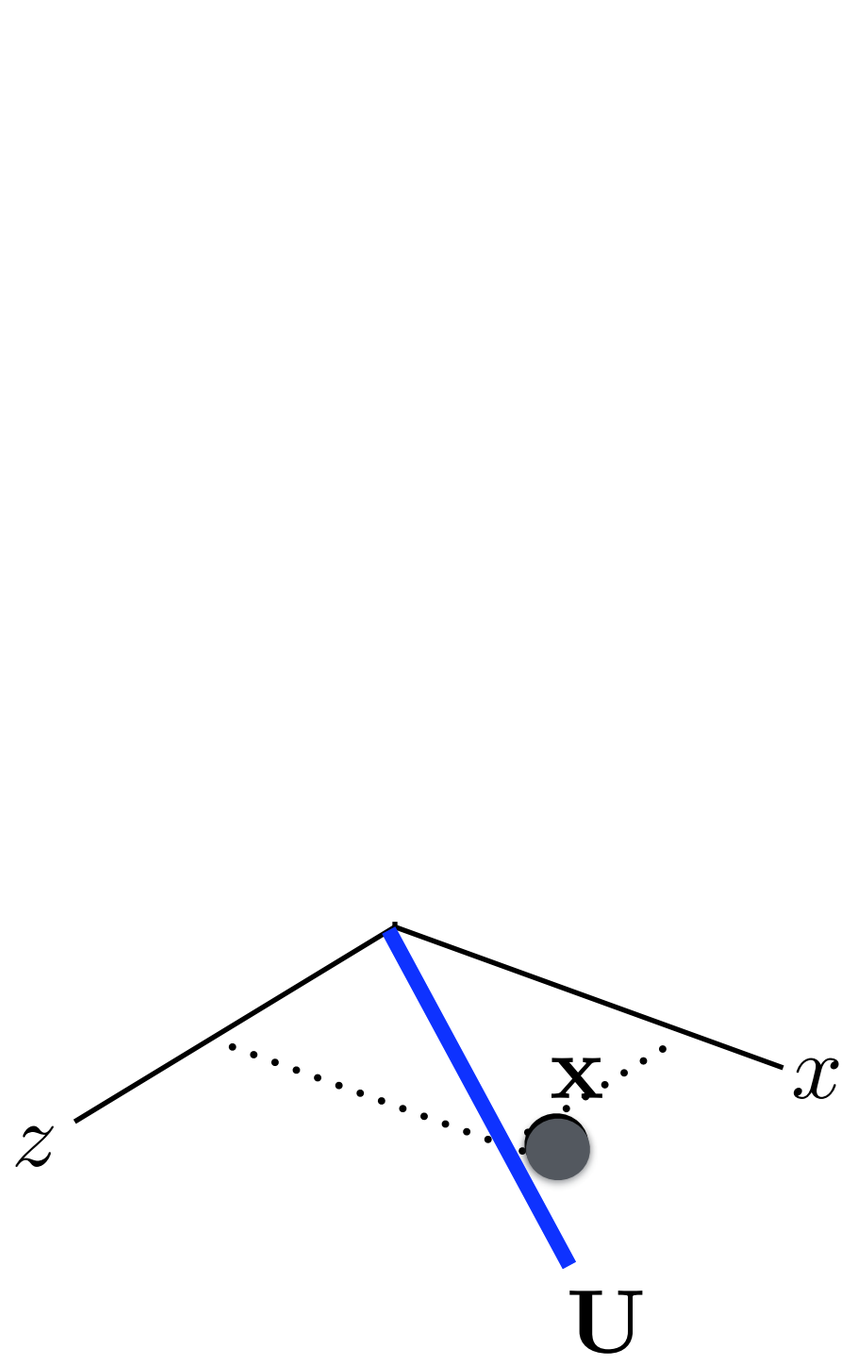
$\mathbf{U}$



new vector

$\mathbf{x}$





Main Idea

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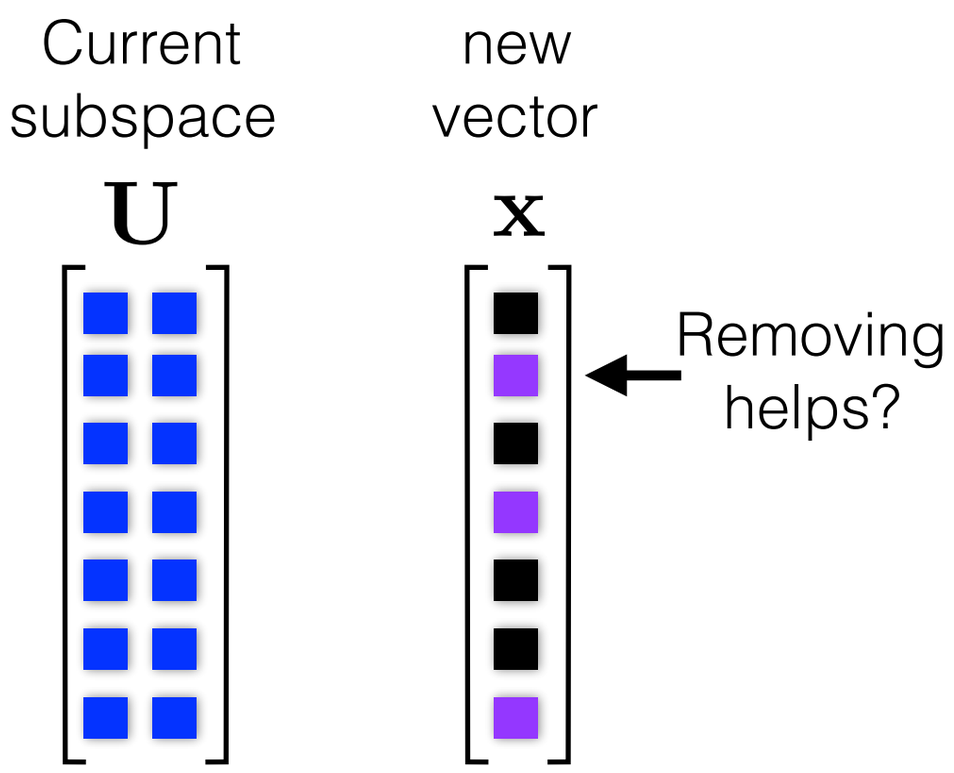

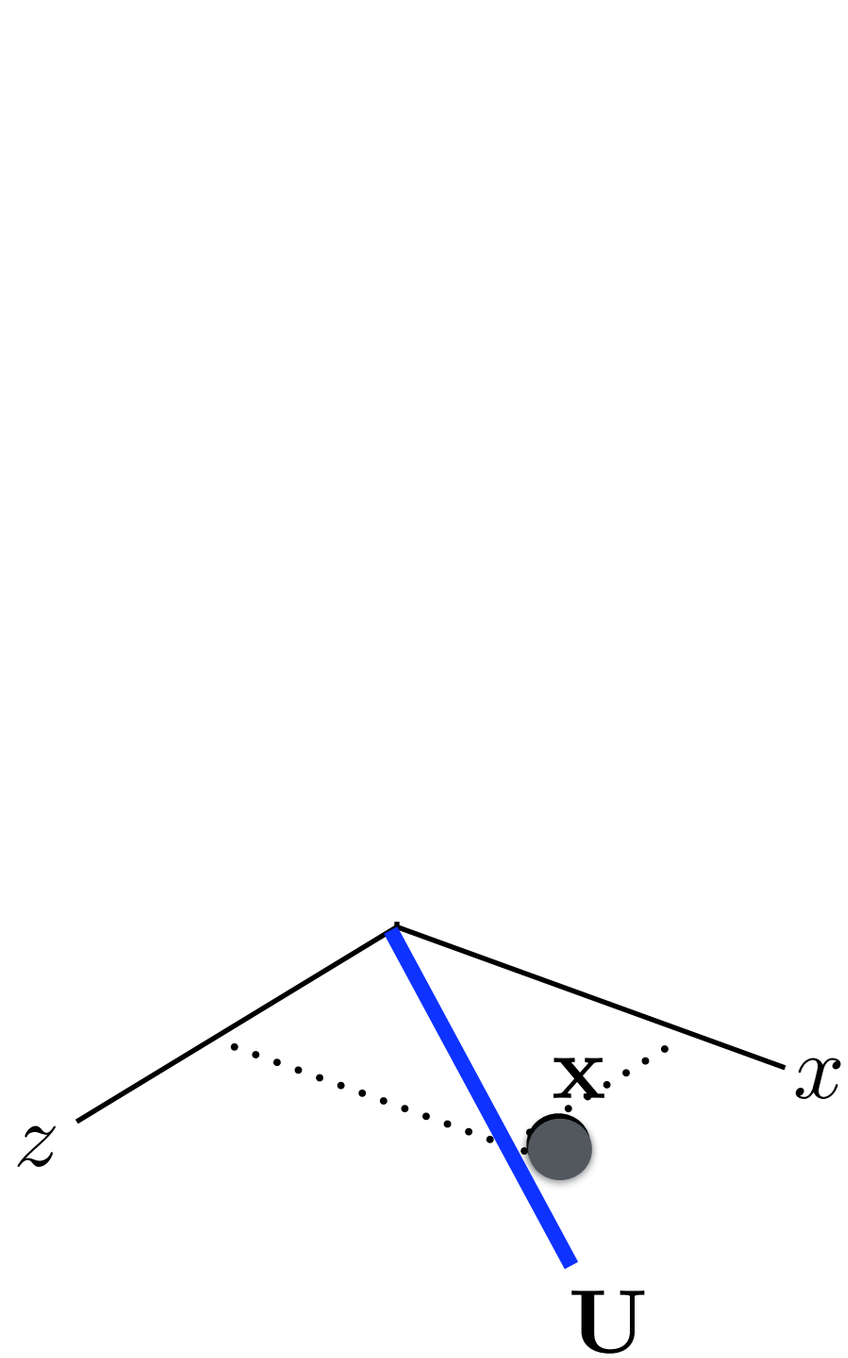

Main Idea

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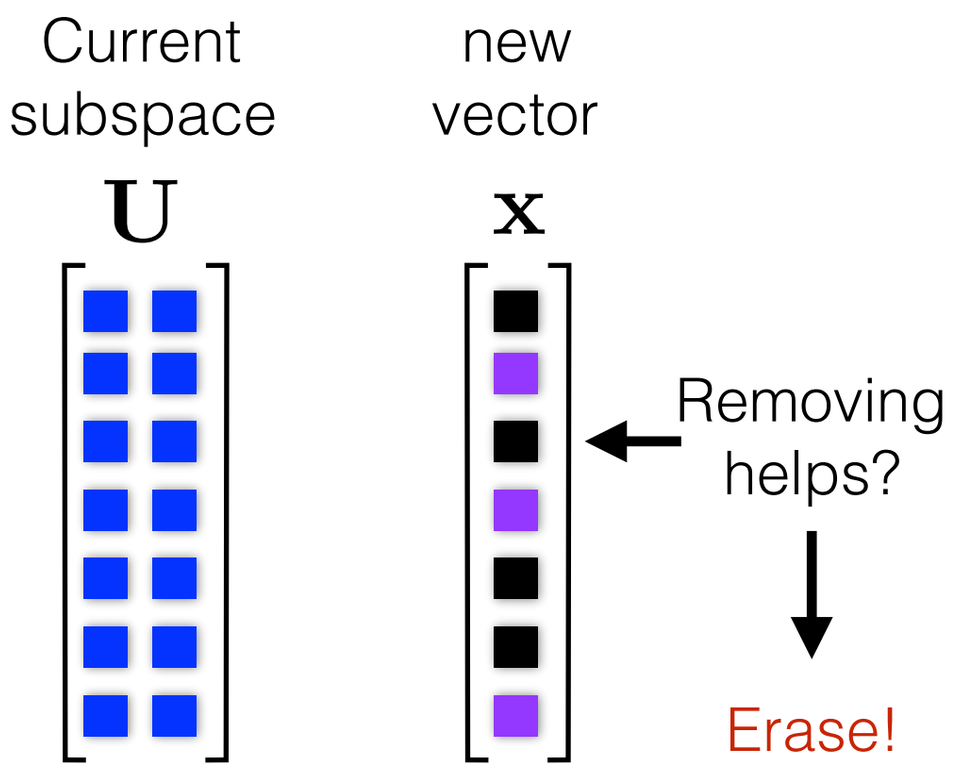

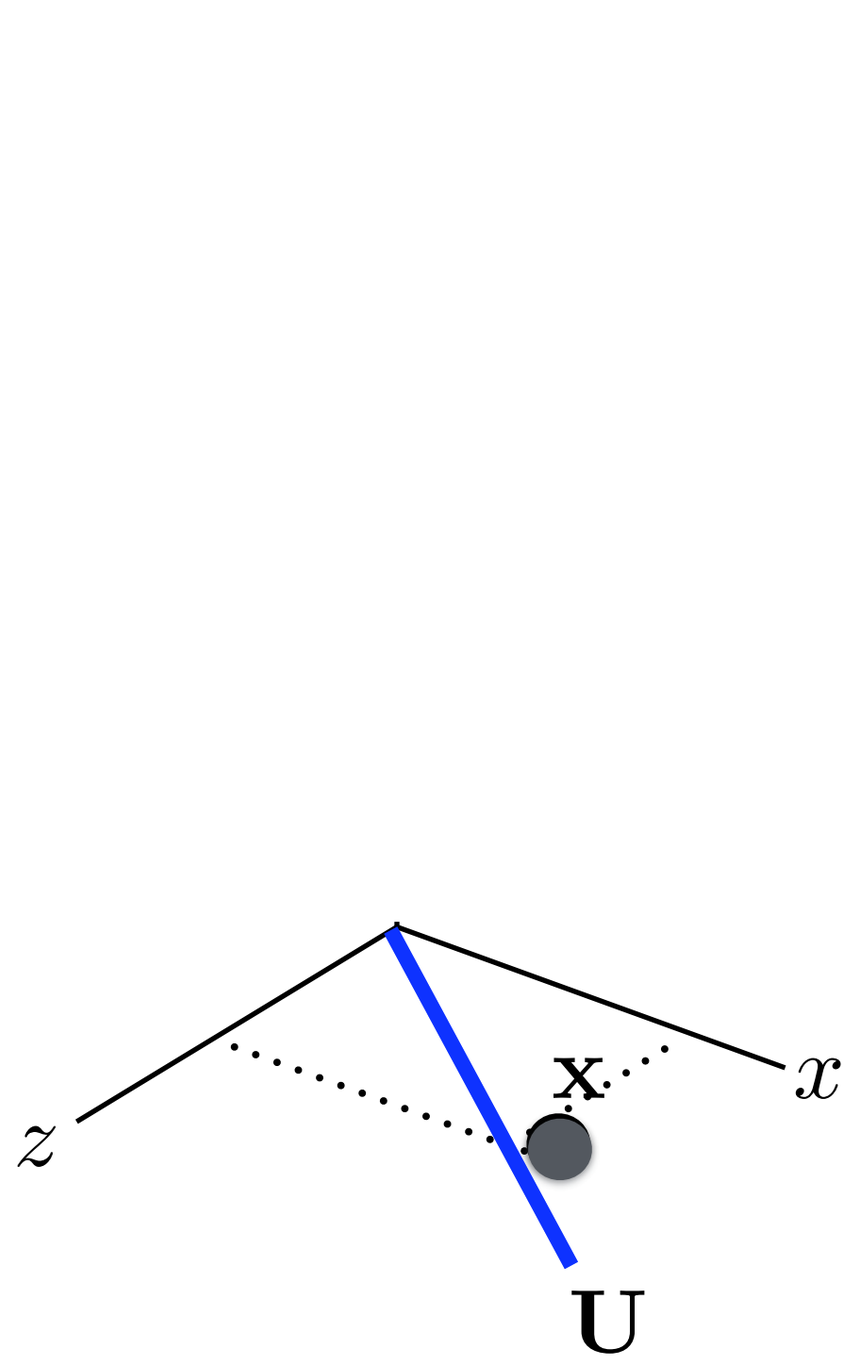

Main Idea

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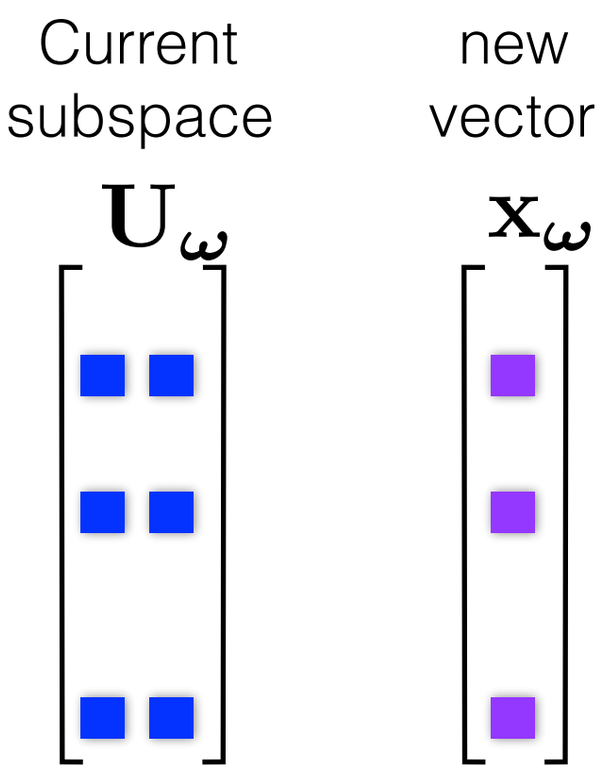
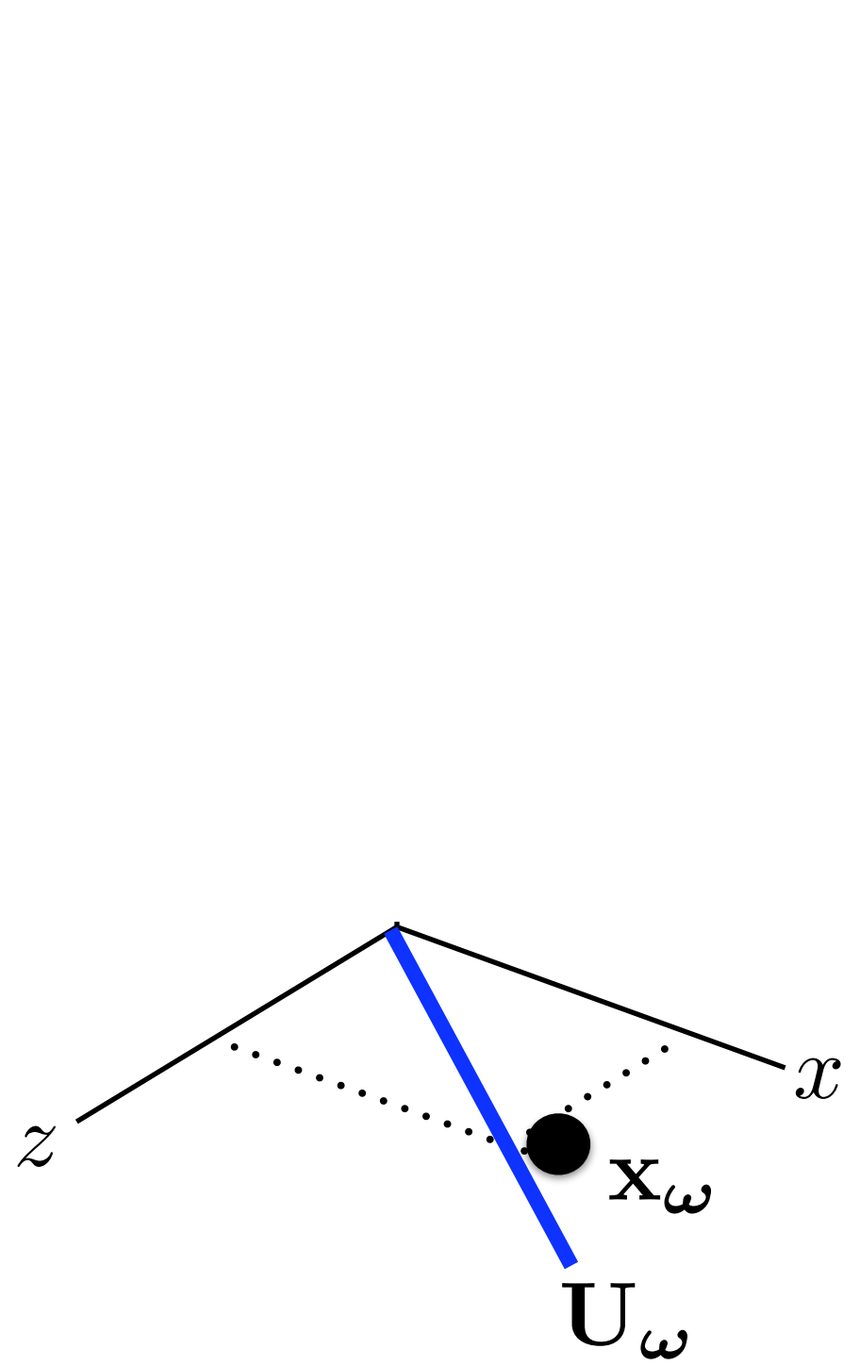
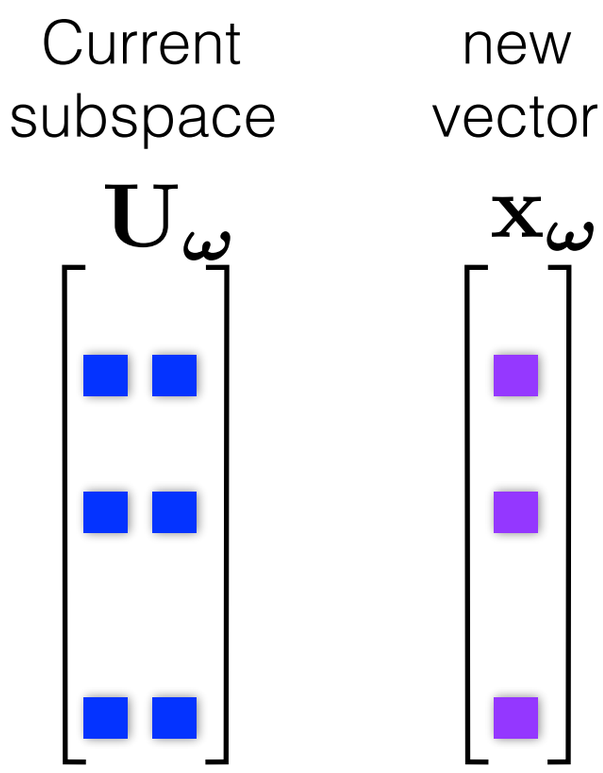




Main Idea

Recover full column



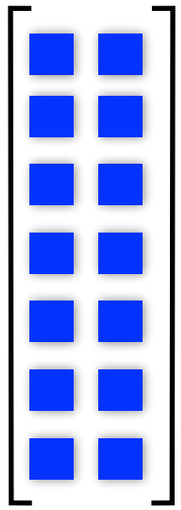
- $\mathbf{U}_\omega \boldsymbol{\theta} = \mathbf{x}_\omega$
- $\boldsymbol{\theta} = (\mathbf{U}_\omega^\mathsf{T}\mathbf{U}_\omega)^{-1}\mathbf{U}_\omega^\mathsf{T}\mathbf{x}_\omega$
- $\mathbf{u} = \mathbf{U}\boldsymbol{\theta}$

Main Idea

Recover full column

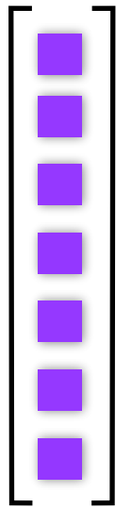
Current subspace

$\mathbf{U}$

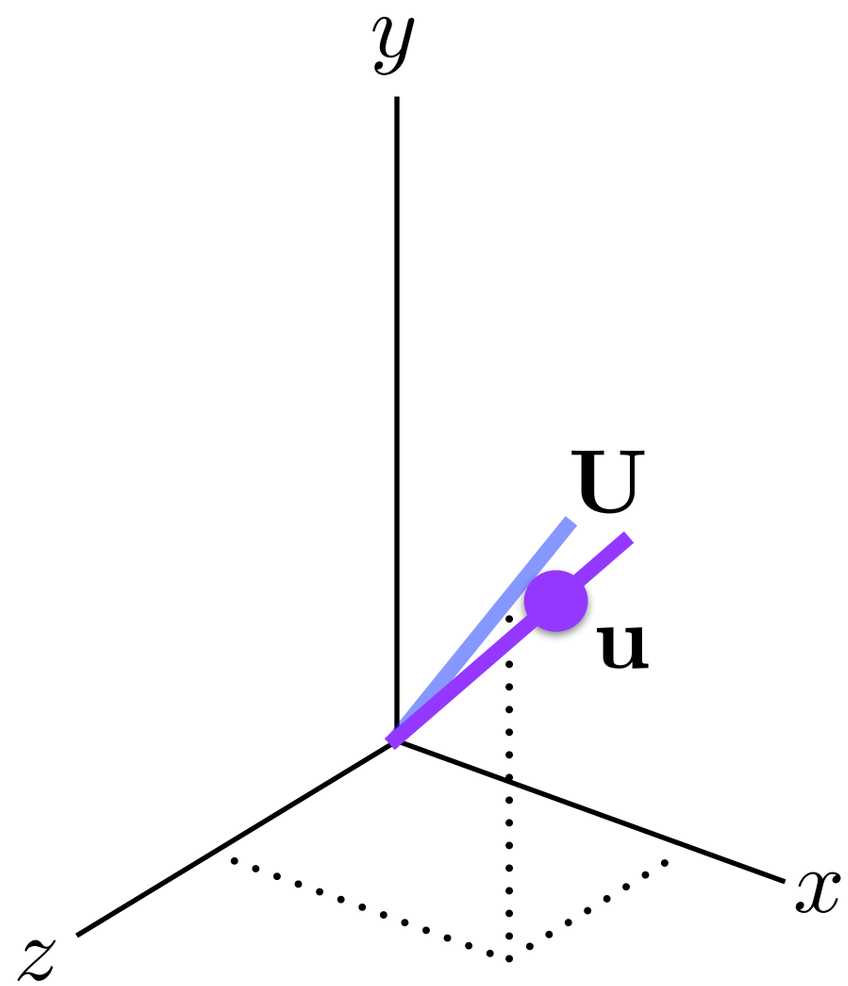


new vector

$\mathbf{u}$



- $\mathbf{U}_{\boldsymbol{\omega}}\boldsymbol{\theta} = \mathbf{x}_{\boldsymbol{\omega}}$
- $\boldsymbol{\theta} = (\mathbf{U}_{\boldsymbol{\omega}}^{\mathsf{T}}\mathbf{U}_{\boldsymbol{\omega}})^{-1}\mathbf{U}_{\boldsymbol{\omega}}^{\mathsf{T}}\mathbf{x}_{\boldsymbol{\omega}}$
- $\mathbf{u} = \mathbf{U}\boldsymbol{\theta}$

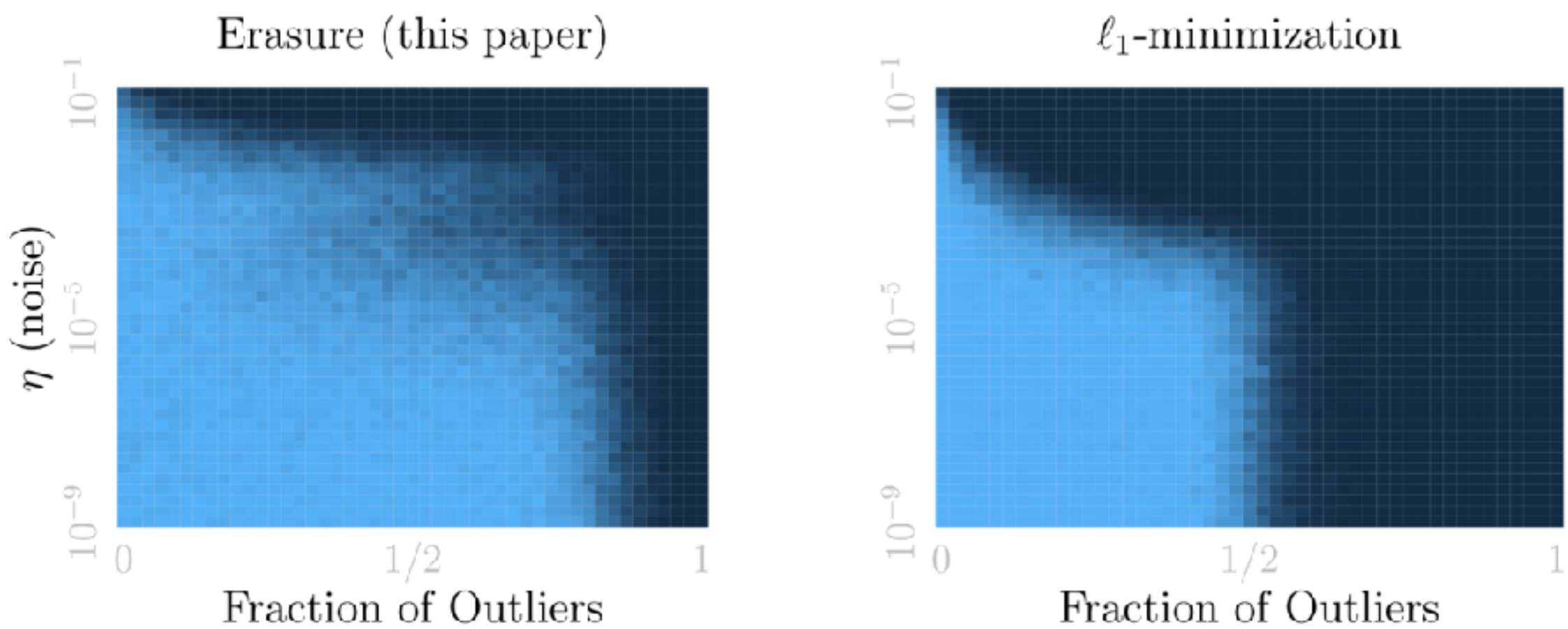


$$\mathbf{U} + \left((\cos(\sigma\eta) - 1)\frac{\hat{\mathbf{u}}}{\|\hat{\mathbf{u}}\|} + \sin(\sigma\eta)\frac{\mathbf{r}}{\|\mathbf{r}\|}\right)\frac{\boldsymbol{\theta}^{\mathsf{T}}}{\|\boldsymbol{\theta}\|}$$

Main Idea

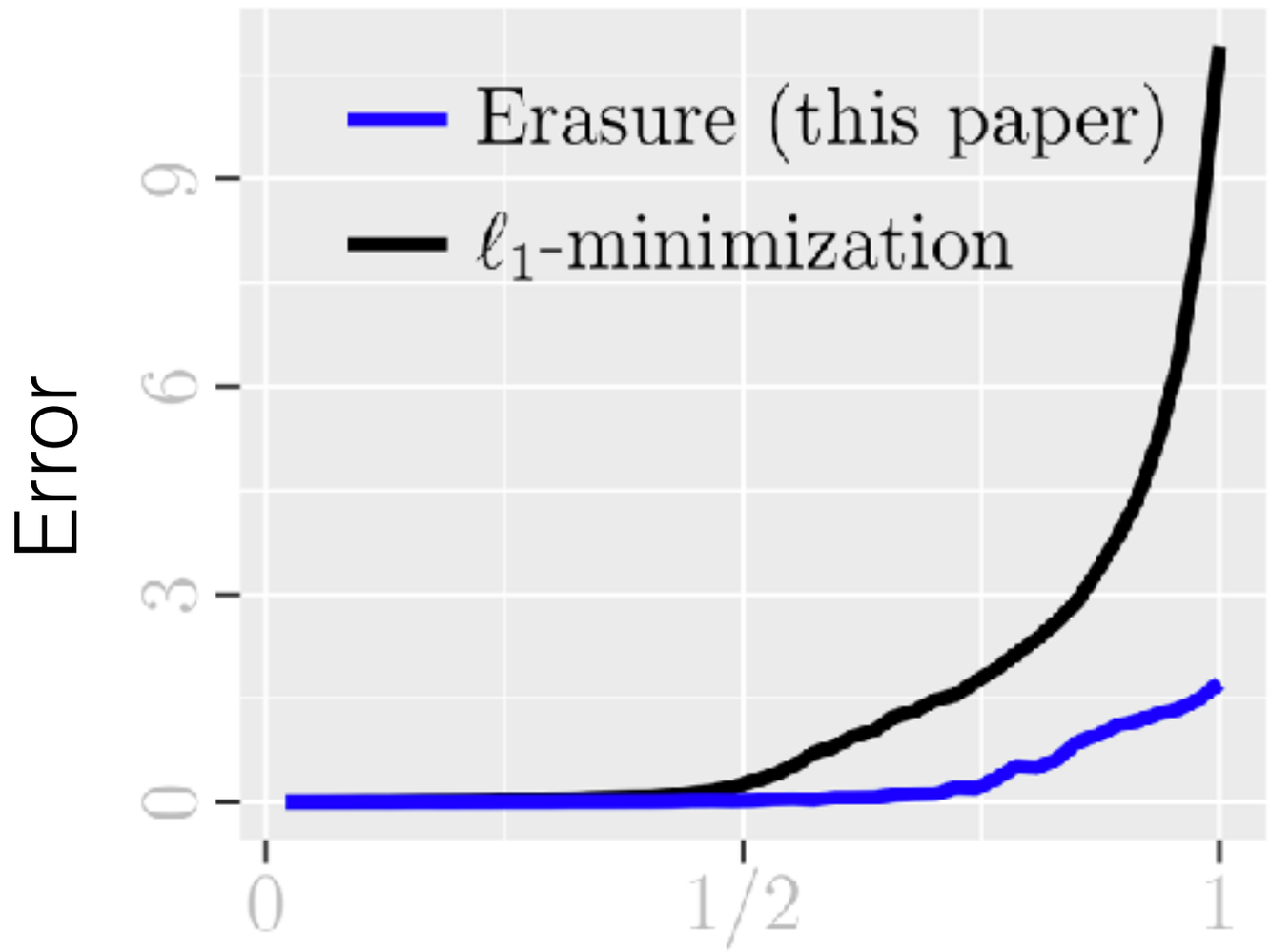
Update Subspace as Before


WOW, AMAZING
PLEASE TELL
ME MORE

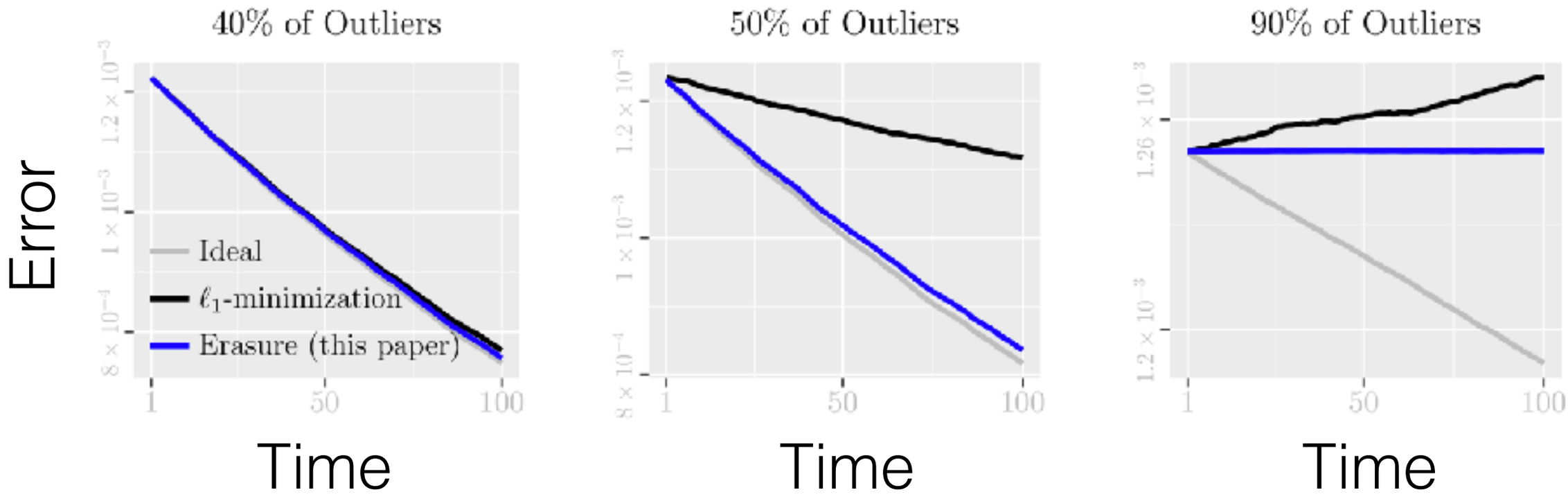


Detecting Outliers

(The lighter the better)



Detecting Outliers

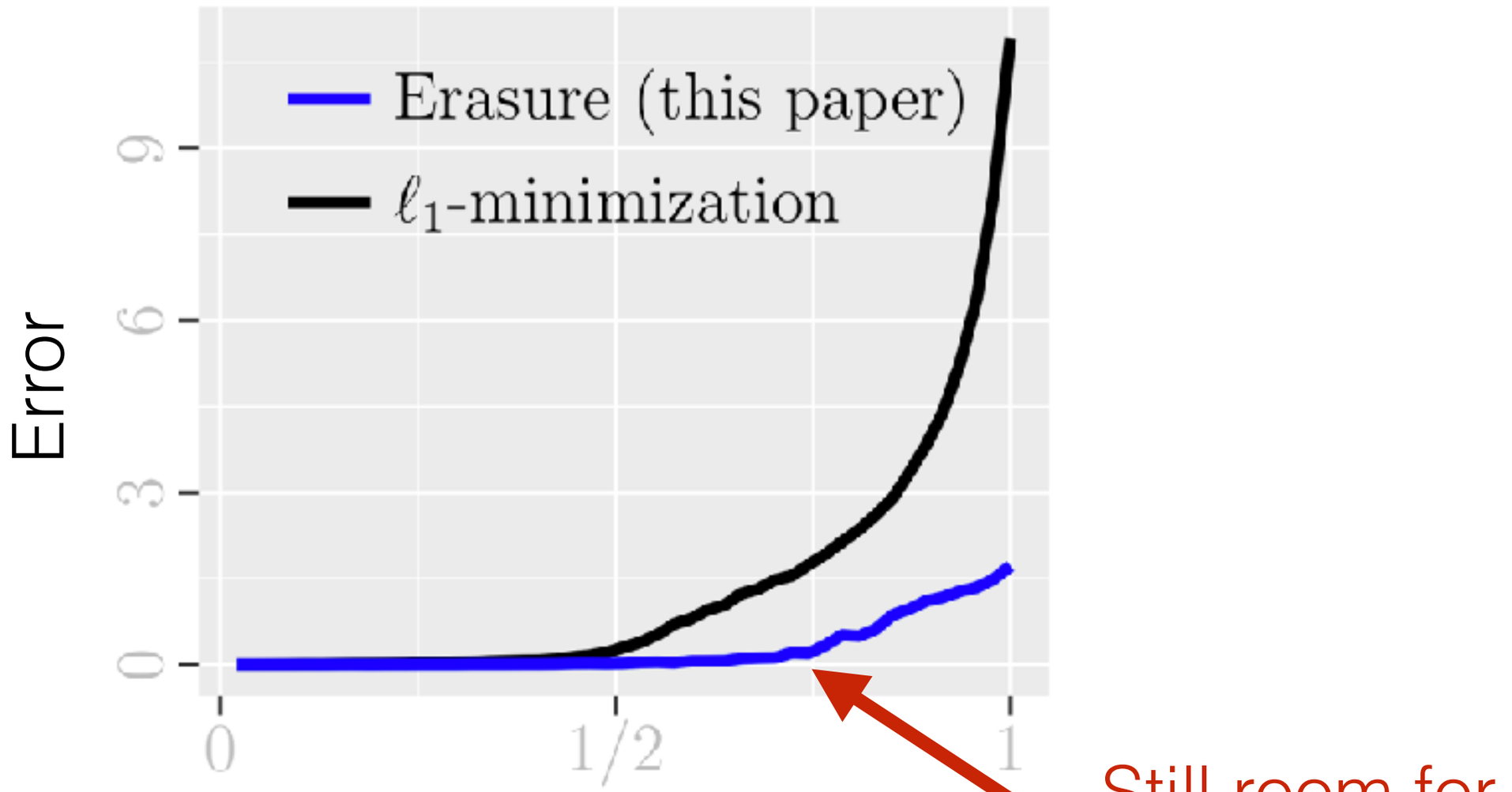


Subspace Tracking

Drawbacks?

No reason why it should work!

Drawbacks?


Erasure (this paper)
ℓ_1-minimization
Error
9
6
3
0
0
1/2
1
Fraction of Outliers
Still room for improvement

Drawbacks?

Thank you!

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