BMI/CS 567: Medical Image Analysis **Spring 2020**

Week 9: Segmentation Evaluation

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9.1 Introduction

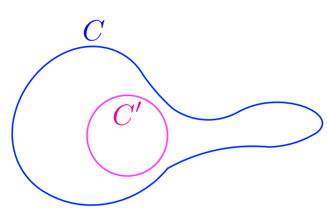
Recall from previous lectures that we have studied several segmentation techniques, such as:

- Thresholding
- Filtering
- Histograms
- Region growing
- Erosion
- Dilation
- Opening (erosion + dilation)
- Closing (dilation + erosion)

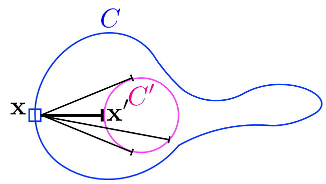
Next we study two options to evaluate a segmentation. The main idea is to compare against a *ground truth* segmentation (e.g., manual segmentation). That is, we compare the results of a segmentation technique against the ground truth on a collection of images, to get an idea of how accurate our technique is.

9.2 Hausdorff Distance

The main idea of the Hausdorff distance is to find the largest minimum distance between two contours. More precisely, let C and C' be two contours (e.g., C is the contour obtained from our segmentation technique, and C' is the ground truth contour).

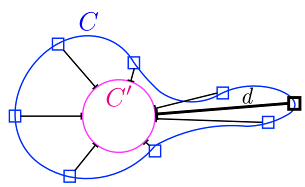


First we start with C. For each point $\mathbf{x} \in C$, we find the closest point $\mathbf{x}' \in C'$:



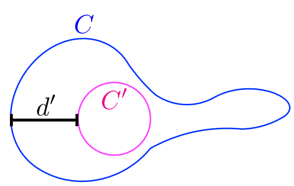
$$\inf_{\mathbf{x}' \in C'} \|\mathbf{x} - \mathbf{x}'\|$$

We repeat this for every point $\mathbf{x} \in C$, and keep the largest distance d:



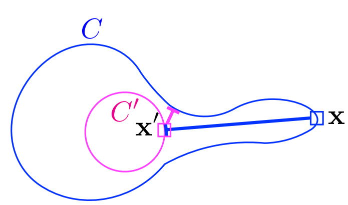
$$\sup_{\mathbf{x} \in C} \inf_{\mathbf{x}' \in C'} \|\mathbf{x} - \mathbf{x}'\|$$

Next we flip the roles of C and C'. That is, for each point $\mathbf{x}' \in C'$, we find the closest point $\mathbf{x} \in C$: We repeat this for every point $\mathbf{x}' \in C'$, and keep the largest distance d':



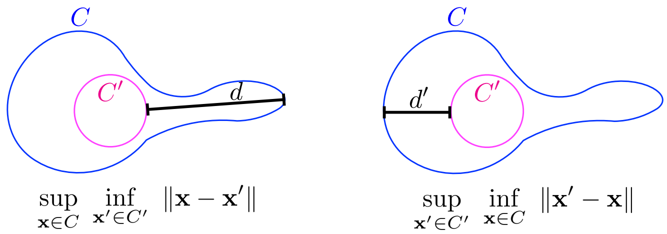
$$\sup_{\mathbf{x}' \in C'} \inf_{\mathbf{x} \in C} \|\mathbf{x}' - \mathbf{x}\|$$

Notice that in general d and d' will be different. For example, here, the shortest distance between $\mathbf{x}$ and C' (depicted in blue) is not the same as the shortest distance between $\mathbf{x}'$ and C (depicted in magenta):



Finally, the Hausdorff distance is the maximum between d and d', which can be summarized in the following expression:

$$H(C, C') \ := \ \max \left\{ \sup_{\mathbf{x} \in C} \inf_{\mathbf{x}' \in C'} \|\mathbf{x} - \mathbf{x}'\|, \sup_{\mathbf{x}' \in C'} \inf_{\mathbf{x} \in C} \|\mathbf{x}' - \mathbf{x}\| \right\}.$$

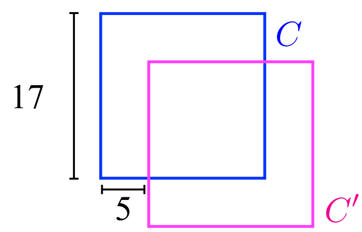


9.3 Dice Coefficient

The main idea here is to measure the relative overlap between two regions, i.e., their common area (intersection) divided by the total area they separately cover:

$$D(C, C') \ := \ \frac{2|C \cap C'|}{|C| + |C'|}.$$

For example, in the next case:



$$D(C, C') \ = \ \frac{2|C \cap C'|}{|C| + |C'|} \ = \ \frac{2(12 \cdot 12)}{17 \cdot 17 + 17 \cdot 17} \ = \ \frac{2(144)}{2(289)} \ = \ 0.498$$