

### Homework 3: Review of Probability

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**Problem 3.1.** Consider a 6-faced fair die.

- (a) What is the probability of obtaining an even number?
- (b) Given that the die rolled an odd number, what is the probability that it was a 3?

**Problem 3.2.** Let  $x$  be a random variable with the following distribution:

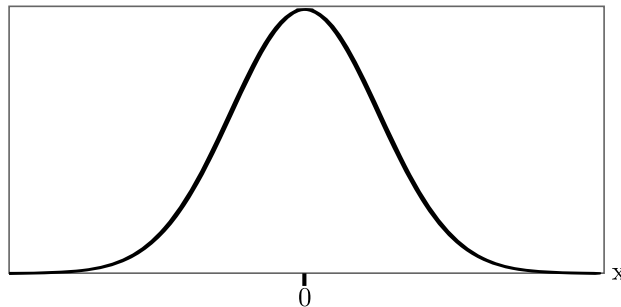
$$\mathbb{P}(x = k) = \begin{cases} 1/2^k & \text{if } 1 \leq k \leq 3 \\ 1/8 & \text{if } k = 4 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Compute  $\mathbb{E}[x]$ .
- (b) Compute  $\mathbb{E}[x^2]$ .
- (c) Compute  $\text{var}(x)$ .

**Problem 3.3.** Recall that the Normal distribution (also known as gaussian) with mean  $\mu$  and variance  $\sigma^2$  is given by:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The following figure shows this density when  $\mu = 0$  and  $\sigma^2 = 1$ . In this same figure, sketch the gaussian distribution with mean  $\mu = 1$  and  $\sigma^2 = 4$ .



**Problem 3.4.** Consider the following samples from an i.i.d. random variable:

Variable	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
$x$	-0.5	-1.6	-1.5	-2.4	2.1
$y$	0.8	-0.6	-2.1	-0.5	2.1

Compute the sample covariance of  $x$  and  $y$ .