# Lecture 15: Expectation

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## 15.1 Expectation

Expected value also know as Expectation. Theoretically it means the return that you can expect for some kind of action. Let X be a continuous random variable and p(x) is the probability density function than the expected value of X is defined by

$$E[X] = \int_x x\, p(x) \tag{15.1}$$

Similarly, Let X be a continuous random variable and g be any function, then the expected value of g(X) is defined as

$$E[X] = \int_x g(x)\, p(x)$$

Intuitively, The expected value of a random variable is the arithmetic mean of that variable, i.e.

$$E(X) = \mu$$

**Example 15.1.1.** If f(x) = x than the expected value of f(x) over all possible value of x is

$$E[X] = \int_x x\, p(x)$$

f(X) is normally distributed with mean $\mu$ and variance is $\sigma^2$ than

$$E[X] = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}} = \mu$$

If mean 0 and variance is 1 then

$$E[X] = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{x^2}{2\sigma^2}} = 0$$

**Example 15.1.2.** To explain ntuitively that expected value also known as mean, lets consider an example, suppose I want to calculate the mean grade of midterm exam of 19 students whose marks are 8,8,9,9,9,9,9,10,7,7,7,7,7,7,7,7,7,7,7 to calculate the average or mean, we do

$$\mu = \frac{sum\ of\ the\ observation}{no\ of\ observation\ (N)} \tag{15.2}$$

so that,

$$\mu = \frac{148}{19} = 7.8$$

The other way to compute average is

$$P = \frac{freqency\ of\ an\ observation}{no\ of\ observation\ (N)}$$

so that,

$$P(8) = \frac{2}{19}$$
$$P(9) = \frac{5}{19}$$
$$P(10) = \frac{1}{19}$$
$$P(7) = \frac{11}{19}$$

So mean also equals

$$\mu = \frac{1}{N} \sum_{x \in \{8,9,10,7\}} x_i\ .\ frequency$$
$$\mu = \sum_{x \in \{8,9,10,7\}} x_i\ .\ \frac{frequency}{N}$$
$$\mu = \sum_{x \in \{8,9,10,7\}} x_i\ .\ P(x_i) \tag{15.3}$$

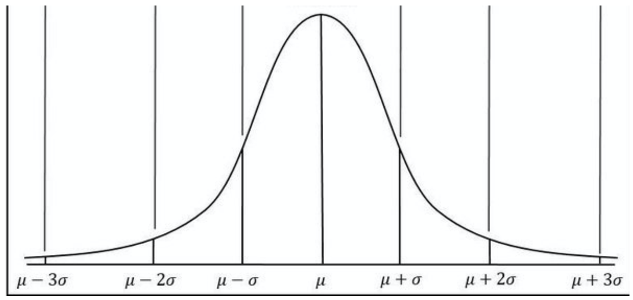
$$\mu = 8 \times \frac{2}{19} + 9 \times \frac{5}{19} + 10 \times \frac{1}{19} + 7 \times \frac{11}{19} = \frac{148}{19} = 7.8$$

so that the result computed from equation 16.1 and 16.2 is same, it means the expected value is equal to mean

**Example 15.1.3.** If $f(X) = (x-\mu)^2$ than the expected value of $f((x-\mu)^2)$ over all possible value of x is

$$E[f(X)] = \int_{-\infty}^{+\infty} (x-\mu)^2\ p(x)dx$$

This is also know as variance, It is the expectation of the squared deviation of a random variable from its mean. Informally, it measures how far a set of (random) numbers are spread out from their average value.




f(X) is normally distributed with mean $\mu$ and variance is $\sigma^2$ than

$$E[(X-\mu)^2] = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}} = \sigma^2$$

## 15.2 Covariance

Covariance is a measure of the joint variability of two random variables. It is defined as the expected product of their deviations from their individual means. Let X and Y are two random variables with means $\mu_x$ and $\mu_y$ then

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)]$$

**Trick 1:**

If X and Y are two independent random variable than

$$E[f(X) \times g(Y)] = E[f(X)] \times E[g(Y)]$$

**Trick 2:**

If X and Y are two random than

$$E[f(X) + g(Y)] = E[f(X)] + E[g(Y)]$$
$$E[f(X) - g(Y)] = E[f(X)] - E[g(Y)]$$

**Trick 3:**

If a is a constant than expected value is also equals to a

$$E[a] = a$$

**Example 15.2.1.** Consider, X and Y are two independent random variable with means $\mu_x$ and $\mu_y$ than

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)]$$
$$Cov(X,Y) = E[(X-\mu_x)] \times E[(Y-\mu_y)]$$

$$Cov(X, Y) = (E[(X] - E[\mu_x)]) \times (E[(Y] - E[\mu_y)])$$

As we know that the expected value of a random variable equal to their mean i.e. E[X] = $\mu_x$ and E[Y] = $\mu_y$

$$Cov(X, Y) = 0$$

## 15.3 Conclusion

This lecture covered Expectation, Variance, Covariance. These are basic components of probability and statistics. Expected value refers to an interesting techniques that is often used in machine learning theory, statistics, probability analysis or during data analysis. Covariance is the relationship and measure the kind of dependence between two variables.