Artificial Intelligence for Sciences and Humanities

Homework 2: Vectors and Matrices

DO NOT POLLUTE! AVOID PRINTING, OR PRINT 2-SIDED MULTIPAGE.

In this homework you will review some basic topics that you need to know in order to understand what is AI and how it works. Each problem is worth 10 points.

Problem 1. Suppose the probability of becoming an influencer is given by the following function:

$$p(x_1, x_2, x_3, x_4, x_5, x_6) \;=\; w_0 + w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6 + w_7x_7,$$

where:

- x_1 represents the number of followers.
- x_2 represents the average number of posts per week.
- x_3 represents the average number of likes per post.
- x_4 represents the average number of comments per post.
- x_5 represents the average of shares per post.
- x_6 represents the total number of saved post.
- x_7 represents the number of users being followed.

Write the equation above in vector form. Specify all the vectors involved in your construction.

Problem 2. Suppose $w_0 = 1/10$, $w_1 = 1/10000$, $w_2 = 1/2000$, $w_3 = 1/90000$, $w_4 = 1/70000$, $w_5 = 1/30000$, $w_6 = 1/20000$, and $w_7 = -1/10000$. Consider a user with the following features: $x_1 = 500$, $x_2 = 3$, $x_3 = 20$, $x_4 = 7$, $x_5 = 4$, $x_6 = 2$, and $x_7 = 1000$. What is the probability that this user becomes an influencer?

Problem 3. Consider another user with the following features: $x_1 = 10000$, $x_2 = 3$ $x_3 = 1000$, $x_4 = 200$, $x_5 = 100$, $x_6 = 50$, and $x_7 = 1000$.

What is the probability that this user becomes an influencer?

Problem 4. What is the problem with the probability of the second user?

Problem 5. Alter the values of the parameters w so that this problem no longer happens. Specify the values of the new parameters w, and the new probability of the second user.

Problem 6. What is the probability of the first user under the new parameters?

Problem 7. The following matrix function encodes the probability of both users:

$$\mathbf{p}^\mathsf{T}(\mathbf{X}) \;=\; \mathbf{w}^\mathsf{T}\mathbf{X}$$

where $\mathbf{p}$ is a vector with 2 elements (each corresponding to the probability of one user), $\mathbf{w}$ is a vector of coefficients, and $\mathbf{X}$ is a matrix containing the information of all users. Specify the entries of $\mathbf{w}^\mathsf{T}$ and $\mathbf{X}$.

Problem 8. Suppose in addition to the probability of becoming an influencer, you also want to predict the probability of earning more than $100,000$ per year from social media, as a function of the same features. Construct a matrix equation that predicts both probabilities simultaneously. Explain all elements of your construction. *Hint: consider making a sketch of your matrix construction.*